第 I 部門

Investigation of Bayesian MCMC estimation for generation of VAR models

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1. Introduction

Vector AutoRegressive (VAR) models are widely used to capture dynamic relationships in multivariate time-series data. In Structural Health Monitoring (SHM), they offer a data-driven framework for detecting changes in structural behavior. While AR coefficients are often estimated using closed-form solutions such as the Maximum a Posteriori (MAP) method with assumed prior distributions[1)], recent likelihood-driven Bayesian approaches using Markov Chain Monte Carlo (MCMC) sampling provide more flexible inference by avoiding parametric assumptions. This study compares these two methods for VAR model evaluation in SHM. Using a Bayes factor (BF) framework[2)] applied to both simulated and experimental data, we assess how each method reflects model fit and sensitivity to structural changes.

2. Bayesian Anomaly Detection

In Bayesian statistics, model parameters are represented as probability distributions, allowing direct assessment of estimation uncertainty. By leveraging this property, probability distributions can be assigned to model parameters in both the baseline (undamaged) state and a potentially altered state, facilitating the computation of the likelihood ratio-based Bayes Factor. As shown in Eq. (1), BF serves as an indicator to evaluate which hypothesis is relatively more plausible based on the observed data, potentially providing a practical criterion for assessing the structural integrity of bridges.

$$BF = \frac{p(D|\mathrm{H}_1)}{p(D|\mathrm{H}_0)} \tag{1}$$

where $p(\cdot\,|\,\cdot)$ represents the conditional probability density function, D denotes the observed feature data, and H_0 and H_1 correspond to the hypotheses that the bridge is in a healthy state and that some anomaly has occurred, respectively.

3. Markov Chain Monte Carlo method

In this study, we alternatively estimate the AR coefficient matrix M in a multivariate autoregressive (AR) model using a Metropolis-Hastings (MH) algorithm, which allows likelihood-based inference without requiring conjugate priors. Each observation is modeled as a linear function of past values and additive Gaussian noise, as shown in Eq. (2).

$$Y = MX + E, \qquad E \sim N(0, \Psi) \tag{2}$$

Here, $Y \in R^{m\times T}$ denotes the observation matrix, $X \in R^{mp\times T}$ contains the inputs, $M \in R^{m\times mp}$ is the AR coefficient matrix, and $\Psi \in R^{m\times m}$ is the noise covariance matrix. At each iteration, a candidate matrix M^* is proposed by perturbing the current state M^t with Gaussian noise as shown in Eq. (3)

$$M^* = M^t + \epsilon, \qquad \epsilon \sim N(0, \sigma^2 I) \tag{3}$$

We compute the log-likelihood of the data given the proposed AR model and noise covariance Ψ, using the multivariate Gaussian likelihood function as shown in Eq. (4)

$$logp(Y \mid M, X, \Psi) = -\frac{T}{2} log \mid \Psi \mid - \frac{1}{2} \sum_{t=1}^{T} (Y_t - MX_t)^T \Psi^{-1} (Y_t - MX_t) \tag{4}$$

The acceptance ratio α is then calculated by comparing the likelihoods at M^* and M^t. The proposed sample is accepted with probability as shown in Eq. (5)

$$\alpha = min(1, exp[logp(Y \mid M^*) - logp(Y \mid M^t)]) \tag{5}$$

4. In-house experiment

A moving vehicle experiment was conducted on a simply supported H beam with an effective length span of 5400 mm, where damage is introduced as the local stiffness reductions from three slits introduced around the bridge's quarter-span as shown in **Fig.1**. Three accelerometers were installed. The condition is

Keywords: Anomaly detection, Bayes factor, Markov Chain Monte Carlo method
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intact when the damages on the bridge are all reinforced, with DMG12 scenario being both damage conditions considered.

5. Results

The difference between the VAR models generated via the MCMC method and the MAP method was evaluated to assess divergence between structural states using Bayes Factors (BFs). **Fig. 2** shows the power spectral density (PSD) computed from the AR coefficient matrices—**Fig. 2a)** corresponds to the MCMC method, and **Fig. 2b)** to the MAP method—highlighting their similar spectral characteristics. **Fig. 3a)** presents the BF results obtained using the AR coefficient matrix from the MCMC method, while **Fig. 3b)** shows the results from the MAP-based approach. Both methods yield similar AR coefficient estimates and produce comparable BF results, indicating consistent detection of structural changes regardless of the inference method used. This suggests that, under the given conditions, both approaches are effective for capturing damage-related variations in the system.

6. Conclusion and Future Work

These results demonstrate that both MCMC and MAP methods produce comparable AR coefficients and Bayes Factor outcomes, effectively identifying structural changes. While the MCMC approach offers a more comprehensive representation of posterior uncertainty, it requires significantly longer computation time. However, in the controlled in-house experimental setting—where external influences are minimal—MCMC convergence was achieved relatively quickly due to the stability of the sampling distribution. This suggests that in such low-variability environments, MAP estimation may serve as a computationally efficient alternative without compromising detection accuracy, whereas MCMC remains preferable when posterior uncertainty plays a critical role. Further research regarding field data is to be expected.

References

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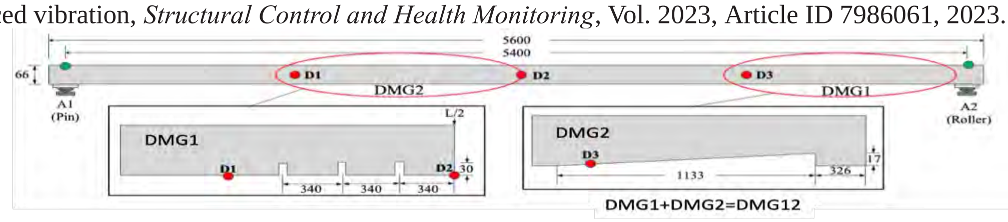

Fig. 1 Layout and Damage scenario of model bridge

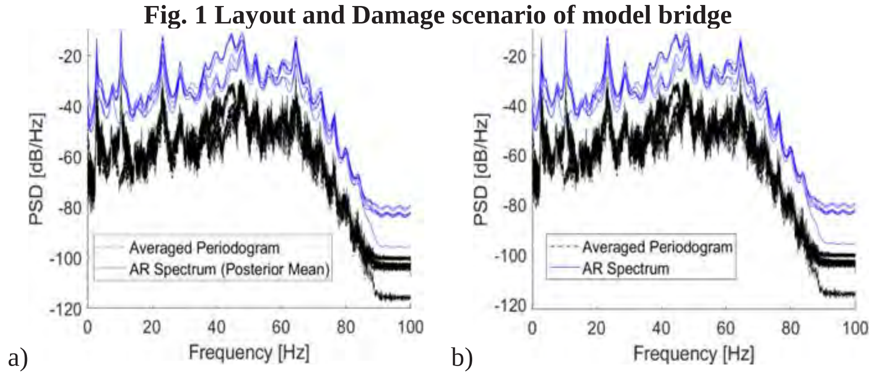

Fig. 2 PSD diagrams from VAR coefficient matrix: a) MCMC method, b) MAP method

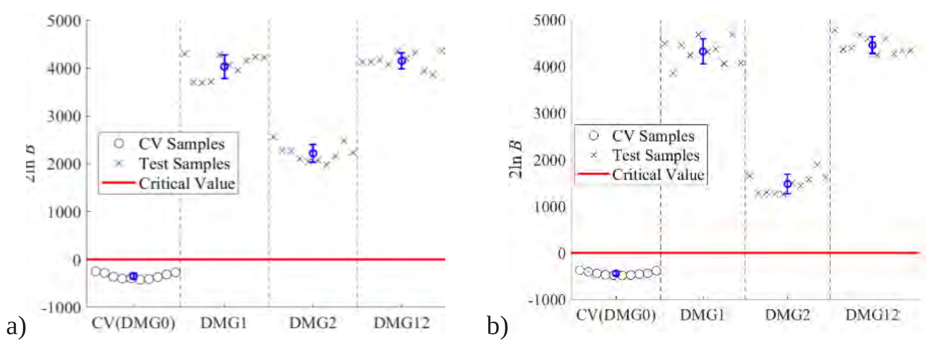

Fig. 3 BF results: a) MCMC method, b) MAP method