第 I 部門　　A copula-ARMA approach for long-term bridge vibration monitoring

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1 Introduction

Gradual deterioration in bridges due to aging has been a critical issue in recent years, resulting in urgent demand for establishing an efficient bridge health monitoring system. Damage in bridge structures is supposed to be diagnosed from the change in bridge dynamic properties, such as frequency and damping constant. However, real bridges are interfered with varying environmental and operational conditions like temperature and traffic. Linear regression models focusing on individual statistical analysis have been widely applied in previous researches.[1)] In fact, commonly existing nonlinear dependency should be taken into consideration. A copula-based method is applied for the first time to compute dependency change between different modal parameters. Copula-based indicators can deal with huge data in multivariate cases, so that the indicator is of great significance for further structural health monitoring and decision making.

2 Real bridge monitoring

The target bridge is a seven-span plate-girder bridge with four accelerometers (UA1, UA2, DA1 and DA2) and two thermometers (T5 and T6) deployed on the first span, as shown in **Figure 1**. The monitoring data consists of three datasets (dataset 1, 2 and 3) obtained from August 5th, 2008 to April 8th, 2015. According to previous researches, the single-variate AR model enables frequencies to be identified from original ambient

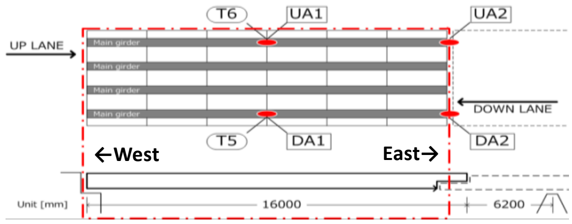

Figure 1 deploying map of sensors.

vibration data measured at each sensor.[2)] The frequencies at the dominant frequencies for each sensor are consequently calculated.

This study focuses on two sensors (DA1 and DA2), starting from elimination of temperature influence of reference data using a linear regression model $f = \alpha T + \beta$ in which f and T stand for frequency and temperature, and α and β are regression parameters. The output becomes a temperature-independent variable.[3)]

3 Copula-ARMA approach

<u>*ARMA model*</u>　ARMA (Auto-Regressive Moving Average) model is acknowledged as a useful tool for predicting future values in time series, composed of both autoregressive part and moving average part. The new value of X_t can be calculate based on previous values of X_{t-i} and the error terms ε_{t-j} with Eq. (1)

$$X_t = \varepsilon_t + \sum \varphi_i X_{t-i} + \sum \theta_j \varepsilon_{t-j} \tag{1}$$

where φ_i and θ_j are parameters.

<u>*Copula-ARMA model*</u>　After best-fit ARMA models are created for two sensors in dataset 1, the copula between residues of two sensors is examined. Copula is a multivariate probability distribution with uniform marginal probability

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distribution of each variable to describe the dependence between random variables. In this study, the Gaussian copula, written as shown in Eq. (2) is chosen because it is the most general and fundamental copula model.

$$C(u_1,\dots,u_N;\rho) = \Phi_\rho(\Phi^{-1}(u_1) + \cdots + \Phi^{-1}(u_N)) \quad (2)$$

where Φ_ρ is the standard multivariate normal distribution function, and Φ^{-1} is the inverse function of standard normal distribution function. ρ stands for a correlation coefficient matrix, and it becomes a scalar in cases of two variables. It conveniently incorporates the correlation into a function that combines each of the marginal distributions to produce a bivariate cumulative distribution function. The copula parameters for dataset 2 and dataset 3 are estimated, utilizing the same ARMA model for dataset 1 (reference data).

To understand fluctuations of parameters better, the copula parameters of three datasets are plotted on the same graph as **Figure 2**. It is not obvious to tell the exact tendency by referring to timeline. The change in the parameter of copula is little, although the parameter of copula does show some increasing tendency, which can be told from the variation in average value.

However there is an interesting finding in dataset 3, where an obvious peak in frequency was detected as shown in **Figure 3** and **Figure 4**. Meanwhile, in the plot of copula parameter, a sudden decline is also observed in **Figure 5**. The obvious peak is caused by drastic changes in traffic during the New Year holidays.

4 Conclusions

This study intends to apply a copula in a long-term bridge vibration monitoring by estimating the nonlinear dependency between two sensors. The bivariate copula model is constructed, and the variation of copula parameter is further analyzed. In spite that computing copula of residues of frequency for the two sensors does not turn out to reach an easily observed indicator as expected, changes in frequencies from two sensors are able to be monitored through copula parameter. Since it is successful in identifying the occurrence of frequency peak, it may become an indicator, which will be of great use in bridge health monitoring in the near future.

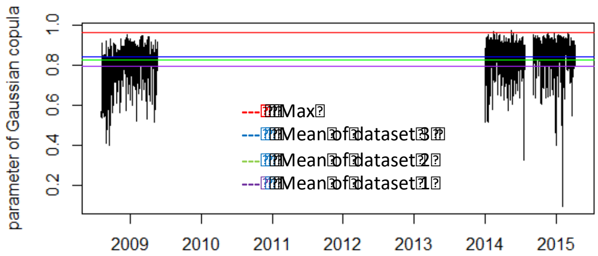

Figure 2 Copula parameter fluctuation with time.

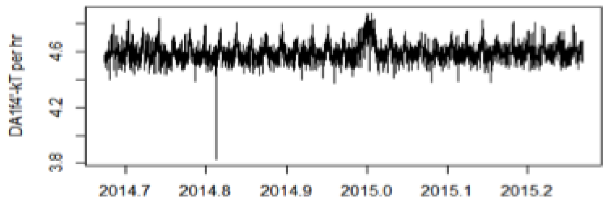

Figure 3 Frequency of DA1.

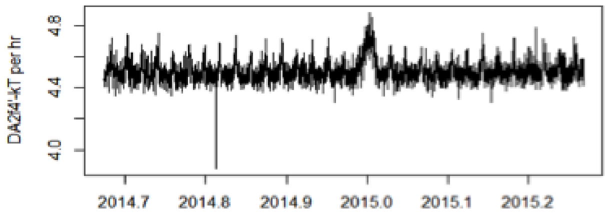

Figure 4 Frequency of DA2.

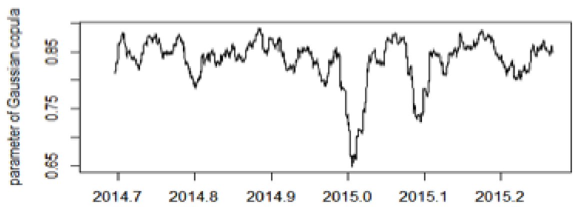

Figure 5 Copula parameter change.

[References]

1) Ko JM, Ni YQ. Technology developments in structural health monitoring of large-scale bridges. Engineering structures, 2005, 27(12): pp. 1715-1725.
2) L. Liung, System identification-Theory for the user 2nd ed., PTR Prentice Hall, Upper Saddle River, M.J., 1999.
3) Kim, et al. Long-term bridge health monitoring focusing on the Mahalanobis Distance of modal parameters, Journal of Physics: Conference Series. IOP Publishing, 2015