## Stage and Discharge Prediction using 2D Dynamic Wave Model with Particle Filter

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## 1. Introduction

River discharge is crucial information in hydrologic fields, and accurate river discharge data is essential for hydrologic model calibration as well. However, there has been little research for quantifying uncertainty in discharge data, and the methods are sparsely applied in practice [1].

In general, river discharge at a certain cross section is estimated from observed water stage and rating curve. And considerable uncertainty is generated during this data conversion. Since water stage data is more reliable than the converted river discharge data, much consideration should be given on the proper usage of water stage data.

This study presents a method to estimate more accurate discharge and reduce the uncertainty of flow condition, using a 2D dynamic wave hydraulic model and a sequential data assimilation concept.

## 2. Methodology

2D dynamic wave model can be a good tool to figure out the flow characteristics in a river channel. The model considers various channel shape, various flow characteristics and wave propagation. However, if the model parameters were fixed, it is hard to reflect the variation of flow characteristics as time passes. Continuous parameter updating during the simulation can reduce the simulation error in this case. In this study, particle filter based Monte Carlo simulation is adopted to merge the various sources and treat the corresponding uncertainties during estimation process.

A simple 2D hydraulic model[2] is utilized in this study to provide the basic river flow simulation. Under an assumption that discharge and flow resistance are not exact, we sequentially assimilated observed water stage data to figure out the right discharge and flow resistance. We adopt two typical methods of particle filter in this study, which are Sequential Importance Sampling (SIS) and Sequential Importance Rampling (SIR) [3][4].

The SIS, which is the traditional algorithm of particle filters, can be described as below:

$$w_t^i \propto w_{t-1}^i \frac{p(z_t \mid x_t^i) p(x_t^i \mid x_{t-1}^i)}{q(x_t^i \mid x_{t-1}^i, z_t)}$$
(1)

$$p(x_t \mid Z_t) \approx \sum_{i=1}^{N} w_t^i \delta(x_t - x_t^i)$$
<sup>(2)</sup>

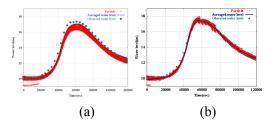
where,  $x_t^i$ ,  $w_t^i$  denote the *i* th particle and its weight, respectively, and  $\delta()$  denotes the Dirac delta function. Weights  $(w_t^i)$  are defined and It can be shown that as  $N \to \infty$ , the approximation (2) approached the true posterior density.

The SIS algorithm thus consists of recursive propagation of the weights and support points as each measurement is received sequentially.

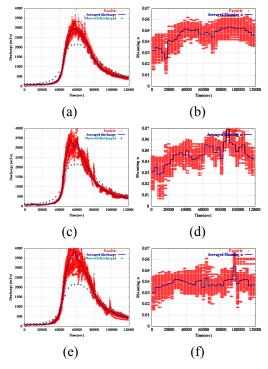
Secondly, the SIR is derived from the SIS algorithm by choosing the importance density and by performing the resampling step at every time index with even weight.

## 3. Results and Discussion

The 2D dynamic wave model is implemented for the Katsura River (total length is about 10km) to estimate



**Figure 1.** Comparison of hourly water stage of SIS result(a) and SIR result(b) at Hazukashi.

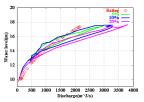


**Figure 2.** Comparison of hourly discharge(a,c,e) and Manning roughness coefficient(b,d,f) at Hazukashi.

more accurate river discharge data from the observed water stage with particle filter algorithm. Upstream condition is given by water stage from a gauging station and downstream condition is given by discharge data that is converted from rating curve.

Figure 1 shows the simulation results from the SIS and SIR algorithm with 100 particles, which include 20% white noise of discharge at initial time step and 20cm observation error of water stage of updating point. The result from SIR algorithm shows better matching with the observed values than the result from SIS algorithm.

More detailed investigation is done with 3 cases of the SIR algorithm. During the resampling procedure in the SIR algorithm, different noise magnitudes for the inlet discharge are given as 5% (a,b), 10% (c,d), 20% (e,f). In



**Figure 3.** The different rating curves according to inlet discharge noise.

Figure 2, the solid line represents estimated discharge and the dotted line represents the observed discharge which is converted from the observed water stage and a rating curve. As shown in Figure 2, the discharge converted from a rating curve tends to smaller than the discharge estimated from the model. We can understand the discharge converted from the rating curve is underestimated.

When the updating noise variation of discharge at every hour is increasing, the fluctuation of manning coefficient is decreased, and vice versa. According to the noise variation of discharge, each rating curve show different tendency (Figure 3). Based on the current results only, it is not easy to decide which rating curve is the most realistic one. Rating curve base data, which is the observation pairs of water level and discharges to develop the rating curve would be helpful to find the most realistic noise magnitude of the inlet discharge. In addition, multi target updating algorithm, which includes additional observation data is under developing for further research.

- 4. Reference:
- Petersen-verleir, A., Soot, A., Reitan, T.(2009) Bayesian rating curve inference as a streamflow data quality assessment tool, Water Resources Management, Vol. 23, No. 9, pp. 1835-1842
- 水理委員会、水理公式集改訂小委員会、水理公 式集例題プログラム集編集部会 (2002) 水理公 式集 例題プログラム集、土木学会
- Peter Salamon, Luc Feyen (2010) Disentangling uncertainties in distributed hydrological modelling using multiplicative error models and sequential data assimilation, Water Resour. Res., Vol.46, W12501, doi:10.1209/2009WR009022.
- 4. B. Ristic, S. Arulampalam, N. Gordon (2004) Beyond the Kalman Filter, Artech House, pp. 35-65