第Ⅱ部門

Uncertainty Assessment of a Conceptual Hydrologic Model by Sequential Monte Carlo Methods

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1. Introduction

Uncertainty in the predictions of science, when applied to the environment, is an issue of great current relevance in relation to the impacts of climate change, protecting against natural and man-made disasters, pollutant transport and sustainable resource management (Beven, 2009). Information of that uncertainty could be used for more credible decision making as well as better understanding of predictions.

Sequential Monte Carlo (SMC) methods are sophisticated model estimation techniques based on simulation. SMC has the advantage of being applicable to non-linear, non-Gaussian state-space models. Over the last few years, the application of these powerful and versatile methods has been increasing, e. g., pattern recognition, weather forecasting, bioinformatics, etc.

Among various hydrologic models, the storage function (SF) model is one of the most commonly used models for flood runoff prediction in Asian countries due to its simple numerical procedure and proper regeneration of nonlinear characteristics of flood runoff.

In this study, we briefly present the theory of SMC as a tool for uncertainty assessment. The SF model is selected and implemented for the middle-sized Japanese catchment. SMC is applied for not only state update but also estimation of parameters simultaneously. Structural inadequacy of the model is analyzed through the time-varying traces of parameters.

2. Methodology

Consider a generic dynamic state-space model which can be described as follows:

$$x_t = f(x_{t-1}, \theta, u_t) + \omega_t \qquad \omega_t \sim N(0, W_t) \tag{1}$$

$$y_t = h(x_t, \theta) + \nu_t \qquad \nu_t \sim N(0, V_t) \tag{2}$$

Where $x_t \in \Re^{n_x}$ is the n_x dimensional vector denoting the system state at time t. The operator $f : \Re^{n_x} \rightarrow \Re^{n_x}$ and $h : \Re^{n_x} \rightarrow \Re^{n_x}$ express the system transition in response to the forcing data u_t, parameters θ. ω_t and ν_t represent the model and the measurement error, respectively.

In the Bayesian recursive estimation, if the system and measurement models are non-linear and non-Gaussian, it is not possible to construct the PDF of the current state x_t given all the measurement analytically. SMC is based on point mass ("particle") representations of probability densities with associated weights (Arulampalam et al., 2002).

$$p(x_t \mid Y_t) \approx \sum_{i=1}^{N} w_t^i \delta(x_t - x_t^i) \tag{3}$$

where x_t^i, w_t^i denote the i th particle and its weight, respectively, and $\delta()$ denotes the Direc delta function.

Since it is usually impossible to sample from the true posterior PDF, an alternative is to sample from a proposal distribution, also called importance density, denoted by $q(x_t \mid y_t)$. The recursive weight updating could be derived as

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follows:

$$w_t^i \propto w_{t-1}^i \frac{p(z_t \mid x_t^i)\, p(x_t^i \mid x_{t-1}^i)}{q(x_t^i \mid x_{t-1}^i, y_t)} \qquad (4)$$

Several variants of SMC methods such as SIR, ASIR, RPF have been developed to overcome the degeneracy phenomenon, selection of importance density and sample impoverishment.

For the unknown parameters, the concept of "artificial evolution" is applied. That means, θ is replaced at each time adding an independent, zero-mean normal increment to the parameter as follows:

$$\theta_t = \theta_{t-1} + \zeta_t \qquad \zeta_t \sim N(0, R_t) \qquad (5)$$

In this concept, the distribution of parameters moves at each time to reproduce the observed data through SMC. Then the fluctuation of parameters and states distribution can be used to assess the uncertainty of the model structure.

3. Preliminary results and discussion

The storage function model was applied to the Katsura river catchment to assess the uncertainty of the model through SMC methods. This catchment is located in Kyoto, Japan and covers an area of 1,100 km^2 (887 km^2 at the Katsura station). Hourly observed precipitation and discharge data were used. Preliminary simulation was performed by Auxiliary Particle Filter (APF) with 1,000 particles. Although the same values of parameters (k=35, p=0.6) were applied, in case of SMC methods, the parameter distributions were evolved when the particles were updated using new observation. Results show SMC methods could enhance estimation and forecasting capability of the SF model compared to deterministic prediction (Figure 1). However, due to structural inadequacy of the model, parameter distribution moved significantly just within the single event (Figure 2).

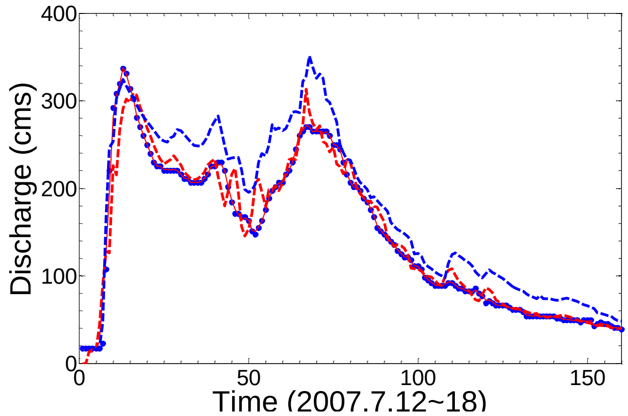

Figure 1. Hourly discharge hydrograph from 12 to 18 July 2007. Red solid and dashed lines represent estimation and 3-hr-lead forecast using SMC, respectively. Blue dashed line represents deterministic prediction without data assimilation. Blue dots denote observed values.

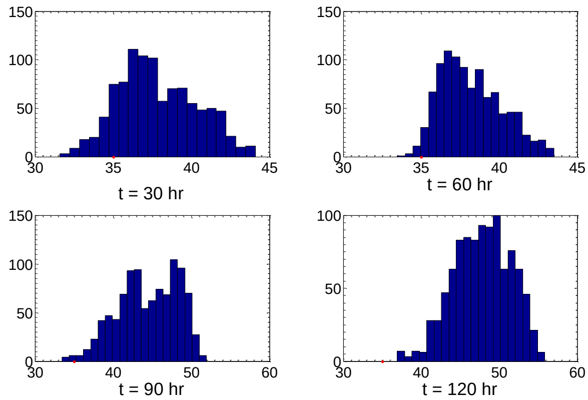

Figure 2. Time evolution of the SF model parameter k distribution at four different time segments

4. Reference:

1. Beven, K. (2009) Environmental modeling: an uncertain future?. Routledge: Oxon.
2. Arulampalam, M. S., Maskell, S., Gordon, N., and Clapp, T. (2002) A tutorial on particle filters for online nonlinear/non-Gaussian Bayesian tracking. IEEE Trans. Signal Process., 50(2), 174-188.