第Ⅱ部門

A Study of the Scale Invariance of Rainfall in Time and Space to Derive Areal Reduction Factors

DPRI, Kyoto University, Student Member ○ Le minh NHAT
DPRI, Kyoto University, Member Yasuto TACHIKAWA
DPRI, Kyoto University, Fellow Kaoru TAKARA

1. Introduction

Many hydrological and meteorological applications require knowledge about the spatial and temporal variability of rainfall over an area. The intensity of point precipitation is only applicable for relatively small areas. For larger areas, design rainfall need to be converted to average area depths. Areal Reduction Factors (ARFs) have been commonly used to obtain this transformation.

In the literature, two different types of ARFs are found (Omolayo, 1993; Sivapalan and Blöschl, 1998): The storm-centered ARFs and the fixed-area ARFs. The storm-centered ARFs are associated with rainfall intensity within the rainfall isohyets of specific storm events, they represent the ratio of average storm depths over an area (defined by rainfall isohyets) and the maximum rainfall depths for the storm (at storm centred). The storm-centered ARFs are used more commonly in PMP (probable maximum flood) estimation. The fixed-area ARFs relate rainfall estimation at point to the average over catchment which is fixed in space. They are estimated by constructing from all available rainfall data at station, the time series of catchment average rainfall, performing the same types of extreme value analyses described above for constructing point IDF curves, and finally relating the catchment rainfall intensities to the point values, for the same return period and duration. This study is concerned with estimating fixed area ARFs.

In hydrological risk analysis and design, one is often interested in the rainfall intensity averaged over a region of area *A* and duration *D*, with return period *T*. Plotting such extreme rainfall intensity I(*A*, *D*, *T*) against *D* for given *A* and *T* produces so-called Intensity Duration Area Frequency (IDAF) curves. For $A \rightarrow 0$ (precipitation at a point), the IDAF curves reduce to the familiar Intensity Duration Frequency (IDF) curves.

For a given location, the ARFs can be defined as ratio between the mean rainfall intensity I(*T*,*A*) and that of point rainfall intensity.

$$ARF(D,A) = \frac{I(D,A)}{I(D,0)} = \frac{IDAF}{IDF} \qquad (1)$$

Some studies have derived the properties of the IDAF curves and the ARFs using non-scaling representations of rainfall. An early attempt in this direction was made by Rodriguez-Iturbe and Mejia, 1974 approach by assuming that the rainfall field is a zero mean stationary Gaussian process. A different approach to ARF estimation, based on crossing properties of random fields, was proposed by Bacchi and Ranzi (1996). Properties of extremes of random functions were used also by Sivapalan and Blöschl (1998). Finally, Asquith and Famiglietti (2000) derived the ARF as the catchment average of the ratios between the *T*-year rainfall depths at distance *r* from the centroid of the storm and at the centroid itself.

In recent years, the concepts of rainfall scale-invariance have come to the fore in both modeling and data analysis in hydrological precipitation research. It opens a new approach to developing a formulation of ARFs. A few studies have assumed that rainfall intensity has scale invariance and used that analysis to derive scaling properties of the ARFs with *A*, *D* and *T*. De Michele *et al*., 2001 have argued directly that the annual maximum value of I(*D*, *A*) could scale in a self similar way with *D* and *A*.

With this approach, this study is to deal with the question how the rainfall properties at a point scale is linked with areal rainfall in terms of time and space. Scaling properties of extreme rainfall in time and space are explored for either disaggregation of rainfall intensity from low to high resolution time scale or aggregation from point to area in spatial scale. Consequently, based on the scale invariance in time and space, ARFs are to be derived.

This study is expected to develop a physical rainfall model that is directly applicable for the Problem of Prediction in Ungauged Basins (PUB).

Le minh NHAT, Yasuto TACHIKAWA, Kaoru TAKARA

2. The scale invariance of rainfall in time and space

A multi-dimension of the scaling properties is described by equation (2). Usually in geophysics, an extension to a multi-dimension framework corresponds to an extension in a space-time domain. Hence, the concept of the scale invariance could be applied in a statistical sense to multi-dimensional space-time random field. The random field *I*(*D*,*A*) exhibits a simple scale invariance behavior if

$$I(\lambda^a D, \lambda^b A) \overset{d}{=} \lambda^{-H} I(D, A) \qquad (2)$$

holds. The equality "$\overset{dist}{=}$" refers to identical probability distributions in both sides of the equations, λ denotes a scale factor and H is a scaling exponent. From (2) comes that: moments of the statistical distribution also scale with a similar power law, the scaling law.

$$E[I^q(\lambda^a D, \lambda^b A)] \overset{d}{=} \lambda^{-Hq} E[I^q(D, A)] \qquad (3)$$

The random field I(*A*,*D*) exhibits a simple scale invariance in a wide sense if (3) holds. If H is a non-linear function of q , the I(*D*,*A*) is a general case of multi-scaling.

From (2) the IDAF is derived and then the IDAF and IDF combined, we obtain the ARFs.

For time scaling, the simple scaling theory can be applied to derive IDF curves consistent with hourly rainfall series in rain gauges where only daily data are available. These curves are developed for gauged sites based on scaling of the generalized extreme value (GEV) and Gumbel probability distributions. Statistical analysis was performed on annual maximum rainfall series for the Yodo River catchment for durations ranging from 1 hour to 24 hours. The results showed that rainfall does follow a simple scaling process in time.

To obtain spatial scaling, spatially distributed rainfall data with 2.5 km^2 spatial resolution and hourly time resolution was arranged for 21 years. By using the data set, the simple scaling properties in space were examined. It was found that two ranges less than 1,000 km^2 and more than 1,000 km^2 show different scaling properties. The rainfall intensity-area-frequency (*IAF*) with fixed duration (*D*) can be derived from a small area to a larger area based on space scaling.

3. Derivation of the ARFs

The IDAF curves reflect the variability of rainfall in time and space, thus it is necessary to make joint analysis of scaling properties of the rainfall field in duration and area (*see figure below*).

We adopt a statistical analysis to obtain the Areal Reduction Factor (ARFs) based on its scaling properties in space and time. The concepts of the statistical scaling are used to study the variability of a random process in time and space. The approach is expected to be more useful and practical to evaluate design rainfall for a specified area. The rain gauges network in the Yodo River basin is the target for this study.

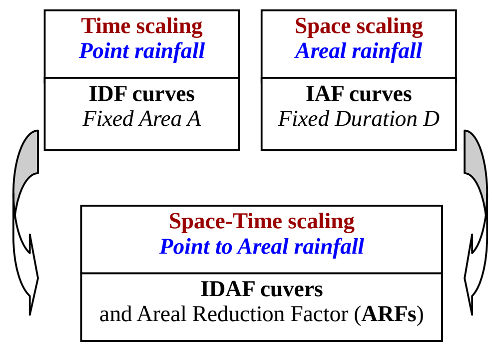

Figure. *Linking the time and space scaling to derive ARFs*

4. References

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