Stochastic Rainfall Fields Modeling using Error Structure of Distribute Rainfall Prediction

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1. Introduction

During short-term rainfall prediction with any prediction model, if an error structure of predicted rainfall is properly analyzed with past prediction results and effective error factors are found, the error structure would be very useful to a present rainfall prediction for giving uncertainty of the prediction values. The important point to consider when we analyze the error structure is that not only amount and distribution of bias between observed rainfall and predicted rainfall (or simply prediction error), but also spatial correlation of the error should be considered.

In this research, error structure of a real-time rainfall prediction by a translation model is analyzed to get its probability distribution and spatial correlation coefficient. And then prediction error fields are simulated as a spatially correlated random field according to the characteristics of the prediction error structure. The simulated error fields are successfully reflect the characteristics of the past prediction error.

2. Error Structure Analysis

Translation model (Shiiba *et al.*, 1984) is used for short-term rainfall prediction. In the translation model, the horizontal rainfall distribution, z(x,y,t)with the spatial coordinate (x,y) at time *t* is modeled as:

$$\frac{\partial z}{\partial t} + u \frac{\partial z}{\partial x} + v \frac{\partial z}{\partial y} = w \tag{1}$$

$$u = c_1 x + c_2 y + c_3$$
, $v = c_4 x + c_5 y + c_6$, $w = c_7 x + c_8 y + c_9$

where, u and v are advection velocity along x and y, and w is rainfall growth-decay rate. Three spatial rainfall distributions, which have 3 km and 5 minutes resolution, are used to determine u and v. When forecasting rainfall fields, the growth decay rate w is always assumed to be zero in this study. Radar rainfall event used here is observed at Miyama radar station of Kinki area on 1990/9/13.

Absolute prediction errors *Ea* at every grid are

calculated from differences between predicted rainfalls Rp, which are predicted at 13:00pm for 30minutes lead time with 5minutes interval, and observed rainfalls Ro for each prediction.

$$Ea = Ro - Rp \tag{2}$$

Frequency distributions of *Ea* show a normal distribution pattern while it is various as time pass (Fig 1). Standard deviations of the *Ea* gradually increase as prediction lead time increase (Table 1), and spatial correlation coefficients also gradually increase (Fig 2). These error characteristics can be admitted as a reasonable result when we consider longer prediction lead time.

Table.1 statistics of the *Ea* (unit: mm/hr)

Leadt	5min	10min	15min	20min	25min	30min
Ave	-0.15	-0.02	0.19	-0.09	-0.32	-0.41
Stdev	4.18	5.56	6.61	7.03	7.56	7.78



Fig. 1 frequency distribution of the Ea





3. Prediction Error Fields Simulation

Let's assume that a predicted error field vector Y can be decomposed to a matrix B which has a spatial correlation characteristic and a random value vector x; Y=Bx. Here, the random values x are uncorrelated each other but follows a given probability distribution. Then a covariance of the vector Y would be same as the covariance matrix of the matrix B. (Tachikawa *et al.*, 2003)

$$\boldsymbol{E}[\boldsymbol{Y}\boldsymbol{Y}^{\mathrm{T}}] = \boldsymbol{E}[\boldsymbol{B}\boldsymbol{x}\boldsymbol{x}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}}] = \boldsymbol{B}\boldsymbol{E}[\boldsymbol{x}\boldsymbol{x}^{\mathrm{T}}]\boldsymbol{B}^{\mathrm{T}} = \boldsymbol{B}\boldsymbol{B}^{\mathrm{T}} \qquad (3)$$

After getting the matrix *B* from the covariance matrix BB^{T} using the *Cholesky* decomposition method, the error field vector *Y* can be simulated with the matrix *B* and the random value vector *x*.



Fig. 4 spatial correlation coerricient compareness

In the figure 3, the simulated error field (dash line) shows same distribution pattern with the distribution of random values x, but the generated prediction error field (prediction + error field, dot line) shows different pattern to the distribution of original error field. It is caused by 'minus rainfall handling' when the prediction fields are generated. If predicted rainfall on any grid becomes minus value after joining with the simulated error field, those minus rainfalls are assumed be zero rainfall because the minus rainfall is not acceptable physically. When distributions are checked again

without minus rainfall handling, the distribution (thick solid line) is matched to the distribution of the original error field.

The same phenomenon is found at the spatial correlation coefficient as shown in the figure 4. From the both results, we can get a conclusion that the simulated error field successfully reflects the error structure of the past predicted rainfall fields.

4. Further Research

There are several problems to be solved and considered. Firstly, as long as the minus rainfall handling affects to the probability distribution and spatial correlation coefficient, more appropriate and reasonable method should be adopted. Secondly, the error structure only with absolute error Ea can not properly simulate error amount difference by rainfall intensity. So, a relative prediction error Er such as;

$$Er = (Ro - Rp)/Rp \tag{4}$$

can be considered. Distribution of the Er in the figure 5 shows that it also has a typical pattern.



Fig. 5 frequency distribution of the Er

Properly simulated prediction error fields would be very useful to figure out uncertainties in a rainfall prediction and also in a runoff prediction.

5. References

[1] Shiiba, M., Takasao, T. & Nakakita, E. (1984) Investigation of short-term rainfall prediction method by a translation model, *Proc.* 28th Annual Conf. on Hydraulics, JSCE, 423-428.

[2] Tachikawa, Y., Y. Komatsu, K. Takara & M. Shiiba (2003) Stochastic modeling of the error structure of real-time predicted rainfall and rainfall field generation, in *Weather Radar Information and Distributed Hydrological Modelling* (ed. by Y. Tachikawa, B. E. Vieux, K. P. Georgakakos & E. Nakakita), IAHS Publ., no. 282, pp. 66-73.