第Ⅱ部門 Scalellogram – A new method of analyzing the hydrologic response of multiscale rainfall field

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INTRODUCTION

Study of multiscale rainfall field is not limited only in understanding the multiscale characteristic of rainfall but also in understanding their hydrological response. The variability of space-time rainfall structure is dependent on the scale of space-time representation, which is responsible to cause input uncertainty in hydrological modeling because the rainfall is one of the major forcing data in the modeling. It is easy to understand that the multiscale rainfall field affects the hydrological analysis having distributed rainfall input. The lumped input-type models also are sensitive to multiscale representation of rainfall as they often use the up-scaled point rainfall data to obtain the catchment average input rainfall. It is necessary to investigate the response of multiscale rainfall field to understand the phenomenon of uncertainty and also to know the proper scale of representation at which the hydrological response is less suffered by the uncertainty.

However, the study of the response of multiscale rainfall is not straightforward due to complexities present in hydrological modeling. There is possibility that the uncertainty present in the model such as in its process representation or the landscape representation may override the input uncertainty. This may lead to dilution of the input uncertainty by either enhancing or damping of overall uncertainty in a way that it might be impossible to understand the role of input uncertainty clearly. In this research, a new method is suggested for analyzing the hydrologic response of multiscale rainfall field, which produce the indicators equivalent to hydrologic response and is called as the Scalellogram plots. The scalellogram plots are able to recognize the properties of the multiscale rainfall structure from the perspective of their hydrologic response, and also able to detect a range of scale at which the rainfall structure becomes dominating cause of the uncertainty in runoff. The experiment is conducted on the Huaihe river basin, China. The distributed rainfall data is the same used by Shrestha *et al.*,[1]. A brief description of the methods and results are presented in this paper.

BACKGROUND

Multiscale fields are often analyzed using spectral analysis method. The spectral analysis is conventionally based on Fourier analysis, in which the signal is compared with a number of basis functions composed of sines and cosines of different frequencies. This kind of analysis is unable to analyze multi resolution data for scale effect investigation[2]. The scale based wavelet analysis is another way of spectral analysis. The wavelet analysis method becomes very complicated when it needs to treat the data having multiple resolutions in space and time, e.g. the case of multiscale rainfall field[3]. The scalellogram is an alternate approach, in which the response coefficients obtained at different scales of representation provides insight of the response due to the multiscale field. The obtained plot of response coefficient for multiple resolution or scale is called as a 'scalellogram', which provides a mean to investigate the consequences of the heterogeneity of spatial signal in a wide range of scale. Studying the relative amplitude of the response coefficients at different scales gives an idea about the dominant scales where the heterogeneity has strong influence in the response function.

METHODOLOGY

This method utilizes the statistical moments of the data distribution. The response coefficient (RC) becomes a function of the statistical moments of the data distribution, which may be written as

$$W_{\bar{x}}(\ ) = f[\mu_1(t), \mu_2(t), \ldots; t]$$

where $W_{\bar{x}}(\ )$ is the RC of rainfall field; t stands for time; $\mu_1(t)$ and $\mu_2(t)$ are first and second moments of the data distribution at particular time t such that

$$\mu_1(t) = \int S(x) p_x(x) dx \quad \text{; at time } t$$

$$\mu_2(t) = \int [S(x) - \mu_1(t)]^2 p_x(x) dx \quad \text{; at time } t$$

It is well known that the statistical moments are the descriptors of the data property. Therefore, the transformed function using the moments have strong basis of reflecting the basic data property in evaluation of the RC, which ultimately may appear in the spectrum to display the proper significance of the data structure.

The RC is supposed to be analogous to catchment response that produces runoff from the rainfall. This is related by an exponent model[4] which is one of the simplest non-parametric models. An experiment with an alternate model may be future research work. Thus an instantaneous RC (IRC) is given by

$$W_l(t) = e^{\mu_1(t)} - 1$$

$$W_d(t) = e^{\mu_1(t)} e^{\sqrt{\mu_1(t)}} - 1$$

where $W_l(t)$ represents the IRC for a lumped spatial data as if $\mu_2(t) = 0$; $W_d(t)$ represents the IRC for a spatially distributed data.

Changing the data scale $\bar{x}$ by upscaling or downscaling process and then re-calculating the RC for the range of spatial scales yield a set of RC for multiple spatial scales. The differences between the multi scale

RC and the reference scale RC produce a scale spectrum of responses called as the scalellogram, which visualize the data property deviation within the multi scale frame relative to the reference scale. This is given by

$$W_{\bar{x},t}(\bar{x}_R)=\int_0^t \left[W_{\bar{x},\tau}(\ )-W_{\bar{x}_R,\tau}(\ )\right]d\tau$$

When the differences between the multiscale RC are observed at a fixed Δ scale interval, it produces a delta scalellogram, which is given by

$$\Delta W_{\bar{x}_i,t}(\ )=\int_0^t \left[W_{\bar{x}_{i+\Delta},\tau}(\ )-W_{\bar{x}_i,\tau}(\ )\right]d\tau$$

The $\Delta W_{\bar{x},t}(\ )$ at a series of i^{th} position of $x+\Delta$ scale gives the delta scalellogram with reference to x scale at the Δ scale interval.

INTERPRETATION OF SCALELLOGRAM

Different structure of scalellogram defines different response characteristics of a multiscale field. These structures can be generalized into five broad types, as shown in Figure 1. These types are named as S1, S2, S3, S4 and S5. Each scalellogram type characterizes the features of multiscale field, which are as follows.

S1: This type of scalellogram shows that the response of underlying field is insensitive to the scale. A uniform response of a multiscale field may produce S1 type scalellogram.

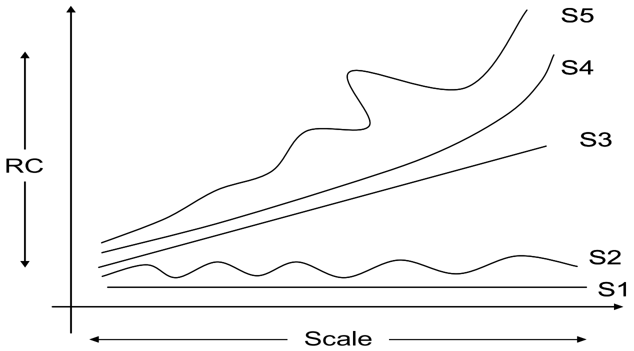

Figure 1 Types of scalellogram representing various response of multiscale field

S2: This type of scalellogram shows that the response of underlying field is insensitive to the scale in a large range of scale (or globally insensitive), but the response may be sensitive to the small change in scale (or locally sensitive). It is hard to say that the S2 type scalellogram represents the scale dependent response of the multiscale field.

S3: This type of scalellogram shows that the response of underlying field is sensitive to the scale and the response is scale dependent. There exist a linear scaling characteristic in the response of multiscale field, which may be defined by laws of simple scaling.

S4: This type of scalellogram also shows that the response of underlying field is sensitive to the scale and the response is scale dependent. There exist a nonlinear scaling characteristic in the response of multiscale field, which may be defined by laws of multi scaling.

S5: This type of scalellogram also shows that the response of underlying field is sensitive to the scale and the response is scale dependent. The response of the multiscale field exhibits a nonlinear and complex scaling characteristic. The complex scaling characteristic may cause difficulty in defining scaling relationships.

Besides these five general types of scalellogram, there may exist some more types. For example, the scalellogram may have globally linear response characteristic (S3 type) but locally nonlinear (S2 type). Further study is advisable to explore all possible types of scalellograms.

RESULT AND DISCUSSION

Different shapes of scalellogram plots are obtained at different catchments, the Suiping (2,093 km^2), Wangjiaba (29,844 km^2) and Bengbu (132,350 km^2). There is a birth of massive dispersion in the scalellogram plots, which corresponds to the initiation of transitional resolution range[5)]. The rise and fall in the scalellogram line occurs due to response of multiscale rainfall.

The fluctuations of scalellogram lines indicate the uncertainty imparted into runoff at different scales of rainfall, which may stand as one prominent understanding of the input uncertainty. This helps to identify a marginal point (Table 1) that separates the critical and non-critical range of scale from the viewpoint of uncertainty. In the non-critical part, the detected uncertainty may be due to the model structure and/or parameters. In the critical part, the uncertainty is more dominated by the rainfall structure than the effect of model structure or parameters.

Table 1 Marginal resolutions (minutes) noticed from the scalellogram.

Resolution (Minute)	Bengbu	Wangjiaba	Suiping
Scalellogram	60	50	20
Delta Scalellogram	60	40	20

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