Analysis on the Flexural Behavior of Externally Prestressed Beam with Large Eccentricity by the Method of Moment-Curvature Relationship

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1. INTRODUCTION

The development of external prestressing has been one of the major trends in constructing and strengthening of bridge beams over the past decade. The ultimate strength of externally prestressed beams is comparatively less than that of the beams in which the bonded tendons are arranged with similar profiles[2,3]. One method of enhancing the flexural strength of the beams is to place the tendons with large eccentricity, by this method, either improvement in strength or amount of prestressing can be achieved. Hamada, et.al has carried out an experimental investigation to study the flexural behavior of such single span structures[4]. It is believed that, by extending this concept to continuous girders, the structural performance can be improved. An experimental study was conducted on two span continuous beams with large eccentricity in reference [1], with emphasis on the influence of tendon layout on ultimate strength and prestressing.

The ultimate flexural analysis of the beam with externally prestressed or unbonded tendon offers two additional difficulties, first, the stress increment of tendon beyond the effective prestress due to applied loading is member-dependent rather than section-dependent, and secondly, there is a loss of tendon's eccentricity in the externally prestressed beam[2]. Some researches have been done to simplify the member-dependent to the section-dependent based on the concept of strain reduction coefficient Ω_u[5]. In this paper, a different method considering material nonlinearity, compatibility of tendon deformation, and loss of tendon's eccentricity is presented.

2. ASSUMPTIONS

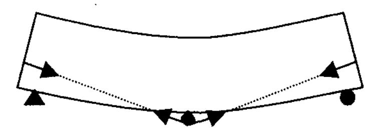

Fig.1 Equivalent loads of external tendon

The following assumptions are adopted in the analysis
(1) Plane section still remains plane after bending.
(2) Bonded tendon, steel and concrete are completely bonded.
(3) The tensile strength of concrete after cracking is neglected.
(4) The friction between external tendon and deviator is neglected.
(5) The beam is in the elastic linear range during prestressing and before cracking.
(6) The stress-strain curves of concrete, steel and PC bar are taken from JSCE Standard (1996).

3. *M-N-* ϕ RELATIONSHIP OF BEAM SECTION

The beam is referred to concrete beam not including external tendon in this paper. The action of external tendon is expressed as the equivalent load applying on the beam as shown in the Fig.1, so the beam is subjected to axial force N and bending moment M, therefore the secondary moment concept is not used in the analysis of external tendon by this method. There are two kinds of loads applying on the beam, one is applied load; the other is tendon's equivalent load. M and N are caused by both applied load and equivalent load. If there is secondary moment caused by bonded tendon in PC beam, in this case, secondary moment is added into M.

The M-N-ϕ relationship of a cracked section is calculated by the discrete element approach. In this approach, the section is divided into a number of horizontal elements, each having the width of the section at that level. For the given curvature ϕ and depth of compressive concrete zone x, summing up the forces and moments of concrete, steel and PC bar, the corresponding M, N are obtained. For M, N known, adjusting different ϕ and x, until equilibriums of force and moment are achieved, then corresponding x and ϕ are got.

4. THE ANALYSIS OF STRUCTURE

After concrete cracking, the stiffness changes along the beam, so the beam must be divided into enough elements to ensure accurate results, especially in the area with a high shear force. It is assumed that the stiffness in each element is a constant and equals to the stiffness at the maximum moment. If the incremental plasticity approach is used, the stiffness of a yielded section is too small, in some cases; it can cause difficulty in calculation and result in unrealistic solution. Therefore the direct iterative approach (successive approximation) is used in this paper (the entire load is applied at one time). The iterative calculation method is as follows

(1) Calculate the equivalent concentrated loads of external tendon prestress force P at deviators and anchorage ends; Calculate the displacements of deviators and anchorage ends due to this equivalent loads by elastic method according to the assumptions.
(2) Calculate M, N and displacements of each element due to the applied loads and equivalent concentrated loads of external tendon prestress force P and its increment ΔP using the stiffness of each element given by step 3 in the previous iteration. ΔP is assumed to be zero and the beam is assumed to be in the elastic range in the first iteration.
(3) Calculate the curvature ϕ using the relationship of M-N-ϕ as shown by point A in Fig.2, if there are secondary moment and axial force due to bonded prestressed tendons, they are added into M and N. The new stiffness is M/ϕ, that is the slope of OA. In the calculation, M is perhaps greater than the ultimate moment of the section, in order to continue calculation, a new small stiffness and depth of compressive zone x are given to this element. Stiffness is relevant to bending moment in statically indeterminate structure. A large original stiffness for an element results in a large moment, so the new stiffness of this element

is small. The actual stiffness is between the original one and new one. Therefor a suitable stiffness between the new one (the slope of OA) and the stiffness used in the previous iteration is chosen as the stiffness for the next iteration. This suitable stiffness can speed the convergence. The choice of value affects the number of iteration, but does not affect the final results.

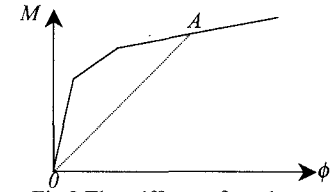

Fig.2 The stiffness of an element

(4) Calculate the strain of external tendon by comparing the displacements of deviators and anchorage ends calculated in step 2 and step 1. According to the PC bar stress-strain relationship, force increment ΔP and corresponding equivalent concentrated loads are obtained, these loads are used in next iteration.

(5) Repeat step 2, 3 and 4 until the following convergence conditions are met

$$\Sigma |\Delta P_i^{(n+1)} - \Delta P_i^n| / \Sigma |\Delta P_i^{(n+1)}| < S \quad \Sigma |R_j^{(n+1)} - R_j^n| / \Sigma |R_j^{(n+1)}| < S$$

in which $\Delta P_i^{(n+1)}, \Delta P_i^n$ are the tendon force increments of ith external tendon for the n+1 and n iteration respectively; $R_j^{(n+1)}$, R_j^n are the stiffness of the jth element for the n+1 and n iteration respectively; S is the given tolerance, S=0.01 in calculation.

(6) Calculate the axial force and bending moment and nodal displacements of the structure.

The method presented in this paper meets the conditions of both M-N-ϕ relationship of the section and compatibility of the longitudinal deformation of external tendon. Because the force of the external tendon is regarded as its equivalent loads and is calculated by comparing the different positions of deviators and anchorage ends before and after loading, so this method can take account the loss of the eccentricity automatically. It can be used generally in any kind of structure with external tendons.

5. THE CALCULATION RESULTS

The symmetrical loading A-1, B-1, C-1 and unsymmetrical loading A-2, B-2 beams[1] are analyzed. The experimental and calculation results are summarized in Table 1, M: Mid span; C: Center support; E: Experiment result; A: Analysis result; L: Left; R: Right. Fig.3 and 4 are load-mid span displacement curves of A-1 and A-2. The calculation results conform well to the experiments, it shows this method is valid and accurate.

Table 1. Summary of experiment and analysis results

No.	Cracking load (KN)				Maximum load (KN)				Ultimate deflection (mm)				Ultimate tendon force (KN)			
	M		C		L-span		R-span		L-span		R-span		L-end		R-end	
	E	A	E	A	E	A	E	A	E	A	E	A	E	A	E	A
A-1	39.2	40.2	36.8	37.1	107.9	121.3	108.6	121.3	82.7	119.1	82.6	119.1	117.1	117.5	116.6	117.5
A-2	34.2	33.8	39.2	40.5	88.1	91.2	47.6	45.6	112.7	92.5	-24.6	-26.0	88.3	86.2	84.4	86.2
B-1	39.6	40.1	37.0	37.1	108.1	122.0	107.6	122.0	110.2	135.2	110.2	135.2	118.3	120.6	118.1	120.6
B-2	36.8	35.2	41.7	41.0	90.8	94.0	49.6	47.0	150.2	102.6	-31.4	-26.6	96.1	89.4	86.3	89.4
C-1	41.7	40.2	36.8	37.2	109.7	119.5	110.9	119.5	80.2	118.2	80.0	118.2	114.1	119.0	115.6	119.0

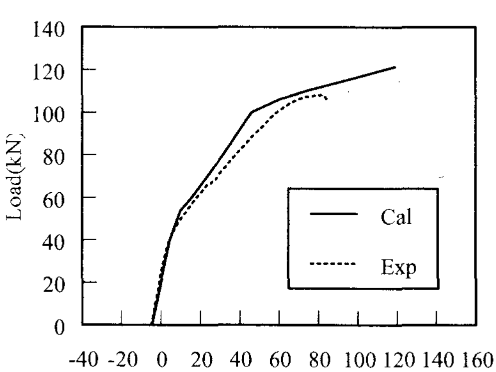

Fig.3 Load-mid span displacement curve of A-1

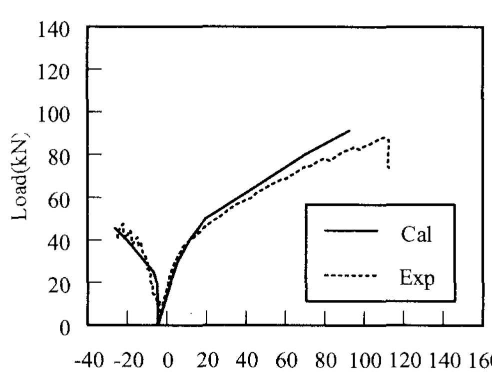

Fig.4 Load-mid span displacement curve of A-2

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