A Cyclic Viscoelastic-Viscoplastic Model for Clays

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1. Introduction

Up to now, many constitutive models for clay have been proposed and studied based on the elasto-plastic or elasto-viscoplastic theory and it has been recognized that the effect of time on the loading process is a salient feature. The linear viscoelastic approach is valid for the behavior in the range of small strains, while viscoplastic modeling of soils is useful in the wide range of strain including failure. In this study, a cyclic viscoelastic-viscoplastic model for clay based on the nonlinear kinematic hardening rule as Oka's model(1992) and Chaboche type non-linear kinematical hardening theory is proposed.

2. Derivation of constitutive equation

Based on the Adachi and Oka(1982) model being a typical overstress model as well as the three parameter model with Voigt element and elastic spring called the linear spring Voigt model, the elastic deviatoric strain rate tensor $\dot{e}_{ij}^{tve}$ is

$$\dot{e}_{ij}^{tve} = \frac{1}{2G}\dot{s}_{ij} + \frac{1}{\mu}(S_{ij} - 2G_2 \cdot e_{ij}{}^{ve}) \tag{1}$$

where, $\dot{e}_{ij}^{tve}$ denotes the total viscoelastic deviatoric strain rate component tensor and it is composed of elastic deviatoric strain rate component tensor $\dot{e}_{ij}^{e}$ and viscoelastic deviatoric strain rate component tensor $\dot{e}_{ij}^{ve}$. G and G_2 is the first and second elastic shear modulus, μ is a coefficient of viscosity, and S_{ij} is the deviatoric stress tensor ($S_{ij} = \sigma'_{ij} - \sigma'_m\delta_{ij}$). Taking into account the deviatoric stress-strain relation, we can obtain the total deviatoric strain rate component tensor included viscoelastic deviatoric strain rate component tensor since $\dot{e}_{ij} = \dot{e}_{ij}^{tve} + \dot{e}_{ij}^{vp}$.

For overconsolidated clay, Oka(1982) developed an elasto-viscoplastic constitutive model based on an overstress type viscoplasticity theory and the non-associated flow rule, and the viscoplastic model for overconsolidated clay was extended to the cyclic model(1992). Considering the nonlinear kinematical hardening rule, static yield funtions are given as follows: For changes in the stress ratio, the static yield funtion used is

$$f_{y1} = \{(\eta^*_{ij} - \chi^*_{ij})(\eta^*_{ij} - \chi^*_{ij})\}^{1/2} - R_{d1} = 0 \tag{2}$$

where, χ^*_{ij} is the kinematic hardening tensor and R_{D1} is the scalar variable.
The evolutional equation for kinematical hardening tensor χ^*_{ij} is given by

$$d\chi^*_{ij} = B_1{}^*(A_1{}^* de_{ij}{}^{vp} - \chi^*_{ij} d\gamma^{vp*}) \qquad \text{and,} \qquad d\gamma_{ij}{}^{vp*} = \sqrt{de_{ij}{}^{vp} de_{ij}{}^{vp}} \tag{3}$$

where, $de_{ij}{}^{vp}$ is the deviatoric viscoplastic strain increment tensor, $A_1{}^*$ and $B_1{}^*$ are material constant and $d\gamma_{ij}{}^{vp}$ is the second invariant of the viscoplastic deviatoric strain increment tensor.
For the first yield function, the plastic potential function is assumed as follows :

$$f_p = \{(\eta^*_{ij} - \chi^*_{ij})(\eta^*_{ij} - \chi^*_{ij})\}^{1/2} + \tilde{M}^* \ln(\sigma'_m/\sigma'_{ma}) = 0 \tag{4}$$

$$\tilde{M}^* = -\frac{\eta^*}{\ln(\sigma'_m/\sigma'_{ma})} \qquad \textit{and,} \qquad \sigma'_{mc} = \sigma'_{mb}\exp\left(\frac{\bar{\eta}^*_0}{M^*_m}\right) \tag{5}$$

in which $\eta^* = \sqrt{\eta^*_{ij}\eta^*_{ij}}$, $\bar{\eta}^*_0$ is the relative stress ratio. In the case of isotropic consolidation, $\bar{\eta}^*_0 = 0$. $\tilde{M}^*$ can be determined by the current stress and σ'_{mc}.
The viscoplastic strain rate tensor $\dot{\epsilon}^{ve}_{ij(1)}$ is assumed to be given by

$$\dot{\epsilon}^{ve}_{ij(1)} = <\Phi_{1ijkl}(F_1)> \Phi_2(\xi)\frac{\partial f_p}{\partial \sigma'_{ij}} \tag{6}$$

where, $<\Phi_{1ijkl}(F_1)> = <\Phi'_{1ijkl}(F)>$ for the case of $F_1 > 0$, and $<\Phi_{1ijkl}(F_1)> = 0$ for the case of $F_1 \leq 0$.
In Eq.(6), $<\Phi_{1ijkl(1)}(F_1)>$ is the functional of F_1, it shows rate sensitivity and can be experimentally determined. $F_1 = f_{y1}$ in which $F_1 = 0$ denotes the static yield function.
In Perzyna's theory(1963), $<\Phi_{1ijkl}(F_1)>$ is dealt with as the scalar function. However, it is assumed that the first material function is the fourth order isotropic tensor function.

$$\Phi_{1ijkl(1)}(F_1) = C_{ijkl}\Phi'_1(F_1) \qquad \textit{where,} \qquad C_{ijkl} = a\delta_{ij}\delta_{kl} + b(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) \tag{7}$$

The concrete shape of the material function is determined referring the previous work(Adachi and Oka,1982, Oka,1982)

$$\frac{\Phi'_{1ijkl(1)}(F_1)}{\sigma'_m} = \exp\left\{m'_o \left\{(\eta^*_{ij} - \chi^*_{ij})(\eta^*_{ij} - \chi^*_{ij})\right\}^{1/2}\right\} \tag{8}$$

where, m'_o is the viscoplastic parameter. In order to take into account the rate independency at the failure state, the second material function $\Phi_2(\xi)$(Adachi, Oka and Mimura 1987) was introduced. This function $\Phi_2(\xi)$ is extended to include the effect of cyclic loading and is given by

$$\Phi_2(\xi) = 1 + \xi \tag{9}$$

where, ξ is the internal variable.

Finally, we obtain a general description of the cyclic viscoelastic-viscoplastic constitutive model for overconsolidated clay.

$$\begin{aligned}\dot{\epsilon}_{ij} &= \frac{1}{2G}\dot{S}_{ij} + \frac{1}{\mu}(S_{ij} - 2G_2 \cdot e_{ij}{}^{ve}) + \frac{\kappa}{3(1+e)}\frac{\dot{\sigma}'_m}{\sigma'_m}\delta_{ij} + C_{01}\frac{< \Phi'_1(F) > \Phi_2(\xi)}{\sigma'_m}\frac{(\eta^*_{ij} - \chi^*_{ij})}{\bar{\eta}^*_x} \\ &+ C_{02}\frac{< \Phi'_1(F) > \Phi_2(\xi)}{\sigma'_m}\left\{\tilde{M}^* - \frac{\eta^*_{mn}(\eta^*_{mn} - \chi^*_{mn})}{\bar{\eta}^*_x}\right\}\frac{1}{3}\delta_{ij}\end{aligned} \tag{10}$$

3. A cyclic viscoelastic-viscoplastic analysis

Using the cyclic viscoelastic-viscoplastic model, we have simulated dynamic behavior of clay. As shown Fig.1, two models appeared different viscoelastic damping characteristics respectively. The proposed model can describe well the energy dissipation compared with a cyclic elastic-viscoplastic model.

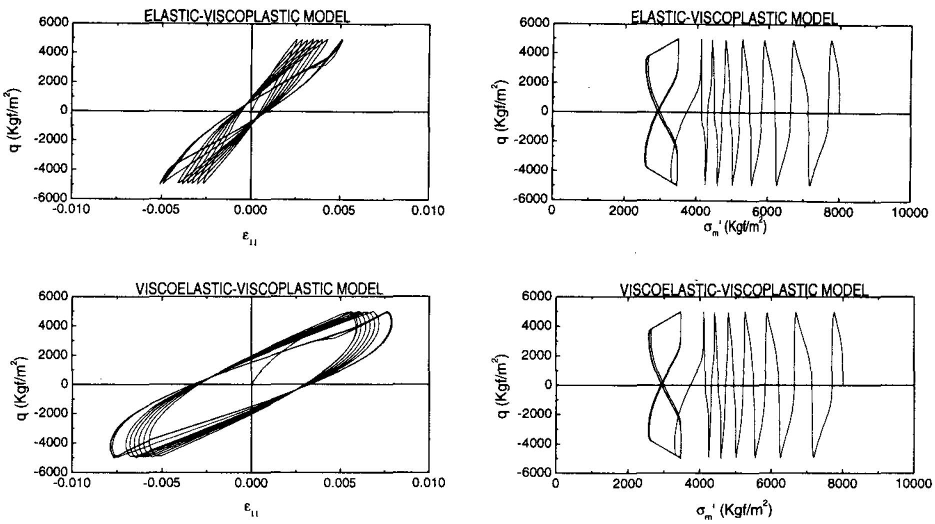

Fig.1 The relationship between a cyclic elastic-viscoplastic model and cyclic viscoelastic-viscoplastic model

Table 1. Material parameters used in the cyclic viscoelastic-viscoplastic analysis

Mean effective stress=0.95(Kgf/cm²),	Consolidation pressure=0.8(Kgf/cm²),	The coefficient of viscosity=2.8E-09		
Rate of deviatoric stress=4.8E-04(Kgf/cm²),	qmax=0.5(Kgf/cm²),	qmin=-0.5(Kgf/cm²),	Time=80,000sec	
Viscoplastic parameters (m_0=12.8, C_1=9.0E-09, C_2=1.5E-08)	Mfc=1.45,	Mfe=1.45,	Mmc=1.2,	Mme=1.2,
Elastic Young's modulus=240(Kgf/cm²),	Second elastic Young's modulue=80(Kgf/cm²),	Initial void ratio=1.922		
Hardening parameters (B_0=57, Bs=15, Bt=1, H=500, G_2=408)	Swelling index=0.0477,	Consolidation index=0.355		

4. Conclusion

In this study, a cyclic viscoelastic-viscoplastic constitutive model for clay has been derived based on the non-linear kinematic hardening theory and an overstress type of viscoplasticity as well as the three parameter model with Voigt element and elastic spring called the linear spring Voigt model. The proposed model is effective to obtain the full formulation of the time-dependent behavior of clay.

Reference

Oka, F.(1992):"A cyclic elasto-viscoplastic constitutive model for clay based on the non-linear hardening rule," Proc. of 4th Int. Sym. on Numerical Models in Geomechanics, pp.105-114.