Network Evolution with Cost-Benefit Investment Rules: A Simulation Approach

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1. Introduction

Transportation systems that provide faster movement of people, goods, and knowledge are an indispensable part of economies and our daily lives. The investments of transportation, by their nature, are expensive, and short run and long run effects on the society and economy that are difficult to measure.

Cost-benefit analysis is the simplest yardstick that can be used to make decisions about the transportation investments. The purpose of this paper is to investigate the evolution process of the transportation network structure under the cost-benefit analysis rule.

2. Assumptions of the Model

The network of cities, connected by a transportation, consists of m cities, indexed by $i = 1, \cdots, m$. The economy produces one fixed product bundle consumed by the population. Perfect competition is assumed to prevail in good markets both within each city and between cities, and commodities are traded without any barriers such as transport costs.

The population is homogeneous and freely mobile among cities, whereas multi-habitation and inter-city commuting are not allowed. Furthermore, the total population of the whole system is given exogenously at any point in time, and is constant.

Each city is geographically monocentric, and consists of two parts, the CBD and residential area. The city residents commute to the single CBD by intra-city railway systems, and pay for the commuting. Land is assumed to be owned publicly. The households achieve the same utility levels regardless of the locations in which they live. The behavior of the household is same as it is given by Kobayashi and Okumura[2].

Production agents employ only the labor force, and local labor force is fully employed in their respective markets. The aggregate production of city i depends on the accessibility to the other cities[1], and is given by

$$Y_i = N_i^{\alpha} \times g(ACC_i)\,,\ 0 < \alpha < 1 \qquad (1)$$

$$g(ACC_i) = \sum_{j=1, j \neq i}^{m} \exp(-\beta d_{ij}) N_j^{\gamma}\,,\ 0 < \gamma < 1 \qquad (2)$$

where the equation (2) defined above gives the accessibility of the city i, N_i is the population of city i, d_{ij} is the transportation parameter between cities i and j, and α,β and γ are parameters.

The central government controls the travel times between cities by improving the existing transportation connections according to cost-benefit analysis. The government does not levy taxes on households and firms when doing this.

3. The Overall Structure of the Model

The model is defined by the following set of equations:

Urban Economies

$$V_i = y_i - c_i \pi^{-\frac{1}{2}} N_i^{\frac{1}{2}} \qquad (\textit{Indirect Utility Level}) \ (3)$$

$$P_i = c_i \pi^{-\frac{1}{2}} N_i^{\frac{3}{2}} / 3 \qquad (\textit{Aggregated Land Rents}) \ (4)$$

$$Y_i = N_i^{\alpha} \sum_{j=1, j \neq i}^{m} \exp(-\beta d_{ij}) N_j^{\gamma} \qquad (\textit{Aggr. Prod.}) \ (5)$$

$$w_i = \alpha Y_i / N_i \qquad (\textit{Wage Rate}) \ (6)$$

$$y_i = w_i + P_i / N_i \qquad (\textit{Household Income}) \ (7)$$

Equilibrium Conditions

$$V_1 = \cdots = V_m = \bar{V} \qquad (\textit{Equilibrium Condition}) \ (8)$$

$$\sum_{i=1}^{m} N_i = N \qquad (\textit{Constant Total Population}) \ (9)$$

In the model described above, N, c_i, and d_{ij} are exogenous variables whereas V_i, u_i, P_i, y_i, and N_i are endogenous variables determined by the equilibrium conditions.

4. Computational Simulations

The network of cities is a simple mesh kind of network which lies on a flat ground. There are n cities on one side of the mesh so that the total number of city nodes is $n \times n$. The city nodes are connected to each other with transportation links. A simple network of 5×5 cities is given in **Figure 1**. The central government improves the existing transportation connections either by improving the quality of the existing connections or introducing a higher-level transportation system. At one stage of the network evolution the government constructs only one link which is selected according to the cost-benefit analysis, which means that the link which provides the highest increase in the equilibrium utility level is selected to be constructed. A link which has been improved once cannot be improved further.

The network is simulated as follows: First, with the given initial populations, the network is brought into equilibrium. After this the travel time parameter, d_{ij}, is improved from its initial value of 1.0 to 0.7. Following the improvement the new utility levels of all of the cities are calculated checked whether they are in equilibrium or not. If the equilibrium is not reached new populations are assigned to each and every city, and the process is repeated until the equilibrium is reached. At one stage of the evolution of the network, this process is repeated for all of the existing links, and the link which provides the highest utility level is chosen as the link to be constructed at that stage. Same procedure is repeated to determine the following links to be constructed.

The convergence to the equilibrium utility level is reached by the following equation:

$$N_{t+1}(i) = N_t(i) + \alpha(u_i - \bar{u}) \qquad (10)$$

which assumes that the population moves toward the locations with above-average utility levels and away from those with below-average. Here $N_{t+1}(i)$ is the population of the ith city to be used in the next iteration step, α is the smoothing parameter which determines the speed of convergence, $\bar{u}$ is the average utility level of the whole city system calculated by taking the simple average of the utility levels of all the cities in the network, u_i is the utility level of the ith city, and $N_t(i)$ is the population of the ith city. The term $\alpha(u_i - \bar{u})$ is the change in the population of ith city, which is positive if u_i is greater than $\bar{u}$ and negative if u_i is less that $\bar{u}$. The population of each and every city is revised accordingly until the whole network reaches an equilibrium of utility.

5. Results

Figure 1 shows the initial equilibrium populations of the cities before any link is constructed. As can be seen from the figure the city at the center of the network becomes the most populated city. Our model is based on the multi-regional static model with costless migration, because of this the city located at the center attracts population. As we move away from the center towards the edges of the network the population of the cities becomes less. Due to this fact, the construction of the links between cities which are highly populated provides the highest utility, and as a result the evolution of the network starts from the center and moves towards the edges of the network. The network evolves with this city remaining at the center. The evolution pattern for the first 20 links is shown in **Figure 2**. In the figure the numbers on the links indicate the order of the construction of the link. **Figure 3** shows the change of utility level as the links are constructed. The figure shows that about 75 % of the maximum utility level which could be achieved by improving all of the links is almost reached when half of the links (20 links) are constructed.

6. Conclusion

This paper investigates the evolution process of transportation network structure with the cost-benefit investment rule. Computational simulations show that evolution with the cost-benefit rules result in formation of highly concentrated network pattern. The evolution starts from the center of the network and evolves towards the edges.

References

[1] Kiyoshi KOBAYASHI: Knowledge Production and Spatial Equilibrium of Firms: Theoretical approach, *Journal of Infrastructure Planning and Management, No.395/IV-9*, 1988

[2] Kiyoshi Kobayashi, Makoto Okumura: The Growth of City Systems with High-Speed Railway Systems, *The Annals of Regional Science*, 1997

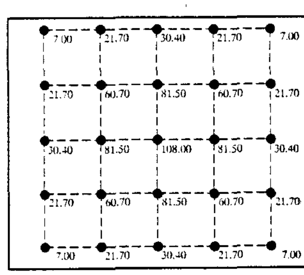

Figure 1. Sample Network of 5x5, and Initial Equilibrium Populations

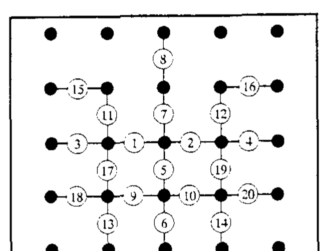
Figure 2. Evolution Process of the Network at the End of 20 Links

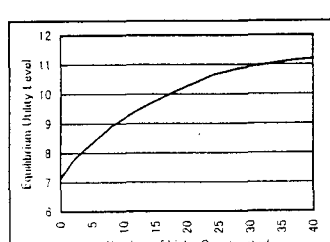

Figure 3. Change of Utility Level w.r.t. the Number of Links Constructed