ALLOCATION OF ENVIRONMENTAL LOADS AMONG POLLUTION SOURCES: A GAME-THEORETIC APPROACH

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1. Introduction

Water pollution problems have intensified through the ages in response to the increased growth and concentration of population and industrial activities in metropolitan regions. The liquid and solid wastes from the comunities generally produce considerable deteriorations in the water bodies, especially those closed ones which tend to accumulate waste loads owing to lack of outlets. To arrive at possible solutions to effective water pollution control, water management authorities have tried to take measures to regulate effluents into closed water systems. Those measures are based on the control of total amounts of pollutants in wastewaters, rather than control of concentration. This solution is called "load control". In Japan, the load control regulations have been specified such that the total COD (chemical oxygen demand) allowance is to be determined for a closed water body and this total load is to be allocated in terms of quotas among the various pollutant sources discharging into the water body [2].

By introducing the concepts of cooperative game theory, this paper places a focus on the above-stated problem of environnmental load assignments among pollutant sources. Application is made to study the problem of Lake Kasumigaura, the second largest lake in Japan. The study illustrates how the problem of load allocation reduces to that of cost-sharing. In allocating the total costs derived from load reduction among the sources the Shapley value method is suggested.

2. Load function formulation and cost allocation

Consider that $n \geq 2$ pollution sources, called players, discharge daily their effluents into a closed water system. One of load functions which represents this process and fits COD load data with reasonable accuracy is the load function $L_i(x)$ defined by [2]:

$$L_i(x) = L_i^0\left(1 - a_i \ln\left(\frac{c_i x}{Q_i^{b_i}} + 1\right)\right),$$

where $L_i(x)$ is the total daily COD load attributable to pollutant source i, L_i^0 the initial load (untread), Q_i the volume of effluent to source i, x the amount spent on effluent treatment, and a_i, b_i and c_i technical parameters depending on local factors.

The objective of control by load is to force dischargers to satisfy the condition $L_i(x) \leq p_i K$. In other words, the total daily COD load $L_i(x)$ to pollutant source i must not exceed the attributable daily COD given by $p_i K$ where p_i is the quota assigned to pollutant source i. Associated with this hold the conditions $p_i \geq 0$ for all $i \in N$ and $\sum_{i=1}^{n} p_i = 1$, where N is the set of players i. In order to achieve the objective of reducing the total COD, dischargers should be encouraged to cooperate with each other if cooperation decreases the total costs. The possible cooperating groups (coalitions) for $n = 3$, so specified in this study, are: the grand coalition which is constituted by all three players and the subcoalitions formed among two players. The cooperative load functions $L_{12}(x)$ and $L_{123}(x)$ which represent the maximum loads achievable at given costs x, by coalition (1,2) and (1,2,3), respectively, are derived as follows:

if $0 \leq x \leq A$:

$$L_{12}(x) = L_1^0\left(1 - a \ln\left(\frac{cx}{Q_1^b} + 1\right)\right) + L_2^0$$

if $x > A$:

$$L_{12}(x) = L_1^0\left(1 - a \ln\left(\frac{L_1^0}{Q_1^b}\frac{cx + Q_1^b + Q_2^b}{L_1^0 + L_2^0}\right)\right) + L_2^0\left(1 - a \ln\left(\frac{L_2^0}{Q_2^b}\frac{cx + Q_1^b + Q_2^b}{L_1^0 + L_2^0}\right)\right)$$

if $0 \leq x < B$:

$$L_{123}(x) = L_1^0\left(1 - a \ln\left(\frac{cx}{Q_1^b} + 1\right)\right) + L_2^0 + L_3^0$$

if $B \leq x \leq C$:

Márcia Maria Rios RIBEIRO, Norio OKADA

$$L_{123}(x) = L_1^0\left(1 - a\ln\left(\frac{L_1^0}{Q_1^b}\frac{cx+Q_1^b+Q_2^b}{L_1^0+L_2^0}\right)\right) + L_2^0\left(1 - a\ln\left(\frac{L_2^0}{Q_2^b}\frac{cx+Q_1^b+Q_2^b}{L_1^0+L_2^0}\right)\right) + L_3^0$$

if $x > C$:

$$L_{123}(x) = L_1^0\left(1 - a\ln\left(\frac{L_1^0}{Q_1^b}\frac{cx+Q_1^b+Q_2^b+Q_3^b}{L_1^0+L_2^0+L_3^0}\right)\right) + L_2^0\left(1 - a\ln\left(\frac{L_2^0}{Q_2^b}\frac{cx+Q_1^b+Q_2^b+Q_3^b}{L_1^0+L_2^0+L_3^0}\right)\right) + L_3^0\left(1 - a\ln\left(\frac{L_3^0}{Q_3^b}\frac{cx+Q_1^b+Q_2^b+Q_3^b}{L_1^0+L_2^0+L_3^0}\right)\right)$$

where, $A = \frac{L_1^0Q_2^b - L_2^0Q_1^b}{cL_2^0}$, $B = \frac{L_1^0Q_2^b + L_1^0Q_3^b - L_2^0Q_1^b - L_3^0Q_1^b}{c(L_2^0+L_3^0)}$ and $C = \frac{L_1^0Q_3^b + L_2^0Q_3^b - L_3^0Q_1^b - L_3^0Q_2^b}{cL_3^0}$.

Exchanging subscripts of $L_{12}(x)$ produces functions $L_{13}(x)$ and $L_{23}(x)$. In these functions it was assumed that the technical parameters a, b and c are the same for all players. It was also assumed that $L_1^0/Q_1^b \geq L_2^0/Q_2^b$, $L_1^0/Q_1^b \geq L_3^0/Q_3^b$ and $L_2^0/Q_2^b \geq L_3^0/Q_3^b$.

Assuming that dischargers reduce load more efficiently and economically by sharing effort, the problem is how to allocate the joint costs among them in a fair manner. The joint cost allocation problem can be modeled as a cooperative cost game [2][3]. The N-person game may be identified by its characteristic function that is a real-valued function v defined on the subsets S. The real number $v(S)$ represents the utility of forming S which is defined as the negative of the least costs achievable by coalition S. For the sake of motivating coalitions, the cost function v so defined must be subbaditive. To be fair, the cost allocation must provide the principles of rationality and marginal costs. The core of the game is constituted by all solutions that satisfy these principles. Another well-known solution is the Shapley value [3]:

$$\phi_i = \sum_{S\subset N, i\in N} \frac{(s-1)!(n-1)!}{n!}\left[v(S) - v(S-(i))\right],$$

where $v(S) - v(S-(i))$ is the player i's incremental cost of adding a coalition S already constructed. The sum is taken over all coalitions S which contains i, and s is the number of players in S.

3. Lake Kasumigaura

Lake Kasumigaura is located at 60 km northeast of Tokyo Metropolitan Region and it presents a drainage area covering 2157 km^2. In the prewar days, the lake basin was predominantly rural and after that the lake basin went through a major postwar rapid urbanization. The water quality of Lake Kasumigaura became increasingly deteriorated through eutrophication. The increased pollution loads were brought in by the rapid population growth, industrial development, increased livestoock production in the lake basin and fish culture in the lake [1].

The western basin of Lake Kasumigaura, Lake Nishiura, which is the largest and most problematic in terms of pollution, is the focus of this study. Fourteen basins are discharging their domestic and industrial effluents into this lake. Development of costs for each of the $2^{14} - 1 = 16383$ possible coalitions of the fourteen basins would be unrealistic. Considering the geographical location, the dischargers are grouped into three big players. Two situations are performed (Figs. 1 and 2, Tables 1 and 2). Situation II comes from a rearrangement of situation I. In these two situations $L_1^0 > L_2^0 > L_3^0$ and $Q_1 > Q_2 > Q_3$ hold. For both cases, a multiple regression analysis of available data yielded the value of local factors as $a = 0.677$, $b = 0.715$ and $c = 0.013$.

Table 1 Situation I - Basic data.

player i	basin area ha	load L_i^0 kg/day	discharge Q_i $10^3m^3/day$
1	50067.4	2601.0	93.5
2	46003.3	1975.9	64.0
3	46529.3	1855.8	60.6
total	142600.0	6432.7	218.1

Table 2 Situation II - Basic data.

player i	basin area ha	load L_i^0 kg/day	discharge Q_i $10^3m^3/day$
1	76883.2	3365.0	118.8
2	46003.3	1975.9	64.0
3	19713.5	1091.8	35.3
total	142600.0	6432.7	218.1

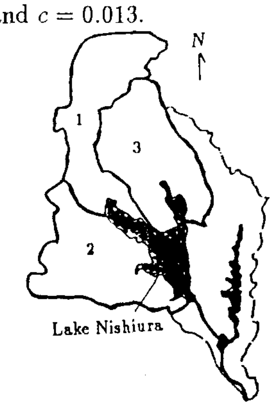

Fig. 1 Players for situation I.

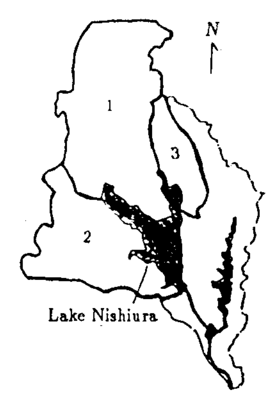

Fig. 2 Players for situation II.

For the permissible total COD load for the lake, six cases are simulated: $K = 6111.1\ kg/day$ (reduction of 5% of the total untreated load); $K = 5789.4\ kg/day$ (reduction of 10%); $K = 5467.8\ kg/day$ (reduction of 15%); $K = 5146.2\ kg/day$ (reduction of 20%); $K = 4824.5\ kg/day$ (reduction of 25%) and $K = 4502.9\ kg/day$ (reduction of 30%). Considering the condition $\sum_{i=1}^{n} p_i = 1$ and the possible regions for each p_i, eg, $1-(L_2^0/K+L_3^0/K) \leq p_1 \leq L_1^0/K$ for player 1, the individual and joint costs for attending each level of K are calculated by using various arrangements of quotas p_1, p_2 and p_3. The costs are given by the characteristic functions $v(S)$ of which values satisfy $v(S) = -L_S^{-1}(Kp_S)$ where the inverse functions $L_S^{-1}(Kp_S)$ are calculated such that $L_S^{-1}(Kp_S) = \min\ (x : L_S(x) \leq Kp_S)$, where p_S is the quota for each coalition S. Seven characteristic functions are identified for Lake Nishiura: v_1, v_2, v_3, v_{12}, v_{13}, v_{23} and v_{123}. Situations I and II are examinated to illustrate how the choice of quotas can affect the final arrangement cost-sharing.

Situation I

In this situation the current level of untreated loads from player 1 is 1.316 times the amount from player 2 and 1.402 times the amount from player 3; player 2 produces loads that are 1.065 times greater than player 3. It may be justified that player 1 pays costs no lower than players 2 and 3 and player 2 pays costs no lower than player 3.

When setting quotas p_i in proportion to current levels of untreated waste loads (called in this study as standard quotas), the individual costs for each player are found to be also proportional to current load levels. These quotas are $p_1 = 0.404$, $p_2 = 0.307$ and $p_3 = 0.289$ for every K simulated. This can be traced back to the load functions $L_i(x)$ and to the amounts Q_i^b that have the same proportion as the current untreated loads L_i^0. For the six cases of K, Table 3 shows the individual costs x_i, the joint costs $v(S)$ and the shapley values ϕ_i performed with the standard quotas p_i.

If standard quotas are set, the individual costs x_i and the Shapley values ϕ_i have the same values (Table 3), which implies that the efficiency of wastewater treatment for the players is the same, irrespective of working together or not. Since there is no player who is more efficient reducer of COD than other, the players may remain working individually at COD reduction.

When quotas p_i are not set in proportion to current levels of waste loads, the costs for each player are calculated as not proportional to current load levels.

Table 3 Situation I - Cost data (million yen) for each K (kg/day).

K	x_1	x_2	x_3	v_{12}	v_{13}	v_{23}	v_{123}	ϕ_1	ϕ_2	ϕ_3
6111.1	153.7	116.5	107.1	270.3	260.6	223.5	277.1	153.7	116.5	107.1
5789.4	316.7	240.8	226.5	557.5	542.9	467.1	783.6	316.7	240.8	226.5
5467.8	492.1	374.5	355.0	866.6	846.8	729.4	1221.3	492.1	374.5	355.0
5146.2	680.8	518.4	493.3	1199.2	1173.8	1011.5	1692.2	680.8	518.4	493.3
4824.5	884.1	673.5	642.4	1557.6	1526.1	1315.6	2199.6	884.1	673.5	642.4
4502.9	1102.9	840.3	802.8	1943.2	1905.3	1642.9	2745.6	1102.9	840.3	802.8

Situation II

Quotas p_i in proportion to current level of untreated waste loads, ie, $p_1 = 0.523$, $p_2 = 0.307$ and $p_3 = 0.170$, lead to individual costs that are not proportional to current load levels (case II-1). In this case, player 1 is the more efficient reducer of COD and its cost share is less than proportionate. The cost data are presented in Table 4 for the case of $K = 5146.2\ kg/day$.

For each K analised there are those quotas which share the costs in the same ratio as untreated loads (case II-2). In case of $K = 5146.2\ kg/day$, these quotas are $p_1 = 0.515$, $p_2 = 0.309$ and $p_3 = 0.176$. In this situation, the core consists of all cost allocations satisfying $x_1 \leq 862.9$, $x_2 \leq 502.9$, $x_3 \leq 281.5$, $x_1+x_2 \leq 1365.8$, $x_1+x_3 \leq 1144.4$, $x_2+x_3 \leq 784.4$ and $x_1+x_2+x_3 = 1647.3$. The Shapley values are shown in Table 4 and it can be checked that cooperation brings a saving of $1647.3-1630.9 = 16.4$ million yen.

As savings of expenditure are brought by any arrangement of quotas analised, it can be concluded that players can reduce COD more efficiently by sharing effort.

Considering the permissible total COD load K for the lake as $5146.2\ kg/day$, Fig. 3 shows the possible quota assignment regions. Six "cost-sharing" regions are defined: region 1 ($\phi_2 < \phi_3 < \phi_1$), region 2 ($\phi_3 > \phi_1 > \phi_2$), region 3 ($\phi_1 > \phi_2 > \phi_3$), region 4 ($\phi_1 < \phi_2 < \phi_3$), region 5 ($\phi_3 < \phi_1 < \phi_2$), region 6 ($\phi_2 > \phi_3 > \phi_1$). Since the current level of untreated loads from player 1 is 1.703 times the amount from player 2 and 3.082 the amount from player 3, it may be justified that player 1 pays costs no lower than players 2 and 3. It also may be required that player 2, which produces loads that are 1.809 greater than player 3 pays costs no lower than player 3. Therefore the region 3, where $\phi_1 > \phi_2 > \phi_3$ may be considered the most fair region. The set of quotas defined by cases II-1 and II-2 are found to be located in this region. Regions 7 and 8 in Fig. 3 are called "no cost-sharing" regions. Any arrangement of quotas in region 7 means that player 3, for which its quota is greater than 0.212, has to reduce more loads than it produces. The region 8, where $p_1 + p_2 > 1$ holds, indicates that players 1 and 2 reduce more loads than required by the water management authority.

The lines labeled ϕ_1=0, ϕ_2=0, ϕ_3=0 indicate, respectively, no cost to player 1, 2 and 3. The lines labeled $\phi_1 = \phi_2$, $\phi_1 = \phi_3$, $\phi_2 = \phi_3$ indicate equal costs for two players. The set $p_1 = 0.556$, $p_2 = 0.299$ and $p_3 = 0.145$ shares the costs equally for the three players and it is located at the central intersection of all lines (point E). The line labeled $p_1 + p_2 = 1$ shows that player 3, for which its quota is zero, has to reduce its current loads to zero level. In this sense he is the most critical player. When the quota set corresponds to point A , the overall load reduction is achieved only by player 1. When it corresponds to point B, the reduction is achieved only by player 2. The overall load reduction is performed by players 1 and 3 when the quota set correponds to point C and by players 2 and 3 when it corresponds to point D.

Table 4 Situation II - Cost data (million yen).

	II-1	II-2
x_1	805.6	862.9
x_2	518.5	502.9
x_3	335.2	281.5
v_{12}	1319.1	1363.4
v_{13}	1112.9	1128.1
v_{23}	843.2	777.9
v_{123}	1630.9	1630.9
ϕ_1	794.2	856.6
ϕ_2	515.7	501.4
ϕ_3	321.0	272.9

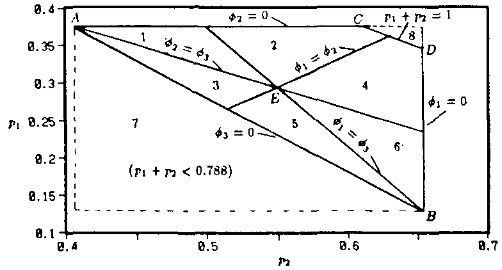

Fig. 3 Situation II - The quota regions for K=5146.2 kg/day.

4. Final remarks

This paper has dealt with the problem of environmental load and cost allocation in western Lake Kasumigaura. Based on its geographical location, two situations have been analised: situation I, where the players have similar amounts of discharge Q_i and loads L_i^0, and situation II, where player 1's Q_1 and L_1^0 are considerably greater. Application of the standard quotas to the first situation has been found fair manner to assign the quotas; since cooperation does not bring any savings, players may remain working individually at COD reduction. For situation II, standard quotas lead to cost-sharings that are not proportional to current loads. In this case, the players can reduce COD loads more efficiently by sharing effort; player 1 appears as the more efficient at load reduction.

In defining the players of the lake system, other aspects (eg, political features) should be considered in further studies. Further allocation of loads and costs among the basins within each group as defined by situations I and II could be modeled by using the game-theoretic approach described, briefly, in this paper.

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