DISPLACEMENT FUNCTIONS FOR SYMMETRICALLY LAMINATED RECTANGULAR PLATES WITH SHEAR DEFORMATION

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1. INTRODUCTION A number of research works have been made for the shear deformable laminated composite rectangular plates in recent years[1,2]. Most of analysis, however, are confirmed within the scope of the rectangular plates simply supported on four edges. This paper describes a development of the displacement functions for the symmetrically laminated rectangular plates based on the first order shear deformation theory. Using the displacement functions obtained, one can solve exactly the rectangular plates with two opposite edges simply supported and the other edges subjected to a wide variety of boundary conditions in the form of Levy-type single series solutions.

2. GOVERNING EQUATIONS Consider a symmetrically laminated rectangular plate of $B_{ij}=0$ and $A_{16}=A_{26}=A_{45}=D_{12}=D_{16}=0$. The governing equations are expressed in terms of the three displacement componentsin the matrix form [1]:

$$\begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \end{bmatrix} \cdot \begin{Bmatrix} \psi x \\ \psi y \\ w \end{Bmatrix} = - \begin{Bmatrix} mx \\ my \\ q \end{Bmatrix} \tag{1}$$

Here, ψx, ψy and w, respectively, are the angular rotations and deflection of the plate; mx, my and q are the surface load terms; and L_{ij} are the commutative, linear differential operators which are given as

$$\begin{aligned} &L_{11}=D_{11}\partial xx+D_{66}\partial yy-\kappa A_{55},\quad L_{12}=(D_{12}+D_{66})\partial x\partial y,\quad L_{13}=-\kappa A_{55}\partial x,\\ &L_{21}=L_{12},\quad L_{22}=D_{66}\partial xx+D_{22}\partial yy-\kappa A_{44},\quad L_{23}=-\kappa A_{44}\partial y,\\ &L_{31}=-L_{13},\quad L_{32}=-L_{23},\quad L_{33}=\kappa A_{55}\partial xx+\kappa A_{44}\partial yy \end{aligned} \tag{2}$$

where D_{ij} and A_{ij} are the plate bending and shear rigidities, respectively; κ is the shear correction factor; and $\partial x=\partial(\ )/\partial x$, $\partial xx=\partial^2(\ )/\partial x^2$, etc.

3. DISPLACEMENT FUNCTIONS Equation (1) is a system of linear partial differential equations with constant coefficients. Thus Heki and Habara's procedure [3] can be used to obtain a displacement function.

3.1 Particular solution *If three functions ϕ_i are the particular solutions of the differential equations:*

$$\det(L_{ij})\cdot\phi_1=-mx,\quad \det(L_{ij})\cdot\phi_2=-my,\quad \det(L_{ij})\cdot\phi_3=-q \tag{3}$$

the particular solutions ψx^p, ψy^p and w^p are given by three displacement functions ϕ_i as

$$\psi x^p=M_{11}\phi_1+M_{21}\phi_2+M_{31}\phi_3,\quad \psi y^p=M_{12}\phi_1+M_{22}\phi_2+M_{32}\phi_3,\quad w^p=M_{13}\phi_1+M_{23}\phi_2+M_{33}\phi_3 \tag{4}$$

In the above, M_{ij} are the cofactors of matrix (L_{ij}) and the explicit form of $\det(L_{ij})$ is

$$\det(L_{ij})=c_1\frac{\partial^6}{\partial x^6}+c_2\frac{\partial^6}{\partial x^4 y^2}+c_3\frac{\partial^6}{\partial x^2 y^4}+c_4\frac{\partial^6}{\partial y^6}+c_5\frac{\partial^4}{\partial x^4}+c_6\frac{\partial^4}{\partial x^2 y^2}+c_7\frac{\partial^4}{\partial y^4} \tag{5}$$

where the coefficients c_i are given as follows:

$$\begin{aligned} &c_1=\kappa A_{55}D_{11}D_{66},\quad c_2=\kappa A_{55}(D_{11}D_{22}-2D_{12}D_{66}-D_{12}^2)+\kappa A_{44}D_{11}D_{66},\\ &c_3=\kappa A_{44}(D_{11}D_{22}-2D_{12}D_{66}-D_{12}^2)+\kappa A_{55}D_{22}D_{66},\quad c_4=\kappa A_{44}D_{22}D_{66}, \end{aligned} \tag{6}$$

$c_5=-\kappa^2 A_{44}A_{55}D_{11}$, $c_6=-2\kappa^2 A_{44}A_{55}(D_{12}+2D_{66})$, $c_7=-\kappa^2 A_{44}A_{55}D_{22}$

For a special case of mx=my=0 and q=q(x),

$$\phi_1=\phi_2=0,\quad \phi_3(x)=\sum_{m=1}^{\infty}(q_m/\Phi_m)\sin(\alpha_m x) \tag{7}$$

where $\alpha_m=m\pi/a$, $\Phi_m=-(c_1\alpha_m^2-c_5)\alpha_m^4$ and q_m is the Fourier coefficient of the load q(x).

3.2 Homogeneous solution If a function ϕ is the solution of the homogeneous part of Eq. (3),

$$\det(L_{ij})\cdot\phi=0 \tag{8}$$

three independent, homogeneous solutions $(\psi x^h, \psi y^h, w^h)_i$ (i=1,2,3) are expressed as

$$(\psi x^h, \psi y^h, w^h)_i=\varepsilon_i(M_{i1}, M_{i2}, M_{i3})\phi \tag{9}$$

where ε_i are arbitrary constants. We then may choose i=3 and $\varepsilon_3=1$, so that ψx^h, ψy^h and w^h become

$$\begin{aligned}\psi x^h&=\partial x\ \{\kappa A_{55}D_{66}\partial xx+[\kappa A_{55}D_{22}-\kappa A_{44}(D_{12}+D_{66})]\ \partial yy-\kappa^2 A_{44}A_{55}\}\ \phi\\ \psi y^h&=\partial y\ \{\kappa A_{44}D_{66}\partial yy+[\kappa A_{44}D_{11}-\kappa A_{55}(D_{12}+D_{66})]\ \partial xx-\kappa^2 A_{44}A_{55}\}\ \phi\\ w^h&=[D_{11}D_{66}\partial xxxx+(D_{11}D_{22}-2D_{12}D_{66}-D_{12}^2)\ \partial xxyy+D_{22}D_{66}\partial yyyy-\\ &\quad-(\kappa A_{44}D_{11}+\kappa A_{55}D_{66})\ \partial xx-(\kappa A_{44}D_{66}+\kappa A_{55}D_{22})\ \partial yy-\kappa^2 A_{44}A_{55}]\ \phi\end{aligned} \tag{10}$$

The simple support conditions of $w=\psi y=Mx=0$ at two opposite sides of x=0, a may be satisfied by the following Levy-type series for ϕ:

$$\phi=\sum_{m=1}^{\infty} Y_m(y)\sin(\alpha_m x) \tag{11}$$

Substituting ϕ into Eq. (8) leads to the sixth-order differential equation for $Y_m(y)$:

$$Y_m^{VI}+f_1 Y_m^{IV}+f_2 Y_m^{II}+f_3 Y_m=0 \tag{12}$$

where $f_1=-(c_3\alpha_m^2-c_7)/c_4$, $f_2=(c_2\alpha_m^2-c_6)\alpha_m^2/c_4$, $f_3=-(c_1\alpha_m^2-c_5)\alpha_m^4/c_4$.

Letting $Y_m(y)=\exp(sy)$, the charactetristic equation of Eq. (12) is obtained as

$$s^6+f_1 s^4+f_2 s^2+f_3=0 \tag{13}$$

or, writting $t=s^2$,

$$t^3+f_1 t^2+f_2 t+f_3=0 \tag{14}$$

This algebraic cubic equation generally has three real roots or one real and two complex conjugates roots. These roots can be easily determined using the method of Cardano. Therefore, the solution form of Y_m will be completly estabulished for each case of the roots. In the case of all of positive real and unequal roots t_i, for example, Y_m takes the following form:

$$Y_m(y)=A_1\cosh(u_1y)+A_2\sinh(u_1y)+A_3\cosh(u_2y)+A_4\sinh(u_2y)+A_5\cosh(u_3y)+A_6\sinh(u_3y) \tag{15}$$

where $u_i=(t_i)^{1/2}$ and A_i are arbitrary constants.

4. CONCLUSION The displacement functions for the symmetrically laminated composite rectangular plates including the effect of shear deformation. The present method could also be extended to find the displacement functions of general anisotropic laminated composite plates for the bending, stability and vibration problems.

5. REFERENCES

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