SHEAR FORCE DISTRIBUTIONS IN ANNULAR SECTOR MINDLIN PLATES

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1. INTRODUCTION

A principal difference between the results predicted with thin and Mindlin plate theories arise in the twisting moment and shear force distributions along and near the edges. The shear force distributions of Mindlin plate of rectangular and skew planforms have been presented [1,2]. This study deals with the bending response of annular sector Mindlin plates with two radial edges simply supported.

2. ANALYSIS

The equations governing the transverse bending are given in terms of displacement components (w,ψ_r,ψ_θ) by the following three uncoupled equations [1]:

$$D\Delta\Delta w = q - \frac{D}{\kappa Gh}\Delta q, \quad \phi = \Delta w + \frac{q}{\kappa Gh}, \quad \Delta\psi - \frac{2}{1-\nu}\frac{\kappa Gh}{D}\psi = 0, \qquad (1,2,3)$$

where $\phi = \mathrm{div}(\boldsymbol{\psi})$ and $\psi = \mathrm{rot}(\boldsymbol{\psi})$, using the vector $\boldsymbol{\psi} = (\psi_r, \psi_\theta)$.

Mindlin's plate equations are six-order differential equations and, therefore three boundary conditions must be specified along each edge. Thus, for example on an edge, θ=constant, these are described as follows:

Simply supported (S): $w=M_\theta=\psi_r=0$, (S'): $w=M_\theta=M_{r\theta}=0$. (4a,b)
Clamped (C): $w=\psi_r=\psi_\theta=0$. (5)
Free (F): $Q_\theta=M_\theta=M_{r\theta}=0$. (6)

The plate considered herein has S-simple support conditions (4a) along the two radial edges. Thus, the Levy-type series solutions have been determined for the plates with arbitrary boundary conditions on the two circular edges and subjected to a variety of surface loads.

3. NUMERICAL RESULTS

In Figs. 1 and 2 are illustrated the distributions of Q_θ along the edge $\theta=0$ and Q_r along the centre line $\theta=\alpha/2$ for the uniformly loaded annular sector plates with S'S'- and FF-boundary conditions on the two circular edges, respectively. The parameters shuch as the inner to outer radii ratio $r_i/r_o=0.5$, the sector angle $\alpha=60°$, the shear correction factor $\kappa=5/6$, and Poisson's ratio $\nu=0.3$ have been used. The results obtained from thin plate theory are represented by dashed lines. The following observations may be made from these figures.

1. S'S'-plate: Figure 1a shows the shear force Q_θ distributions. It follows from this figure that Mindlin theory agrees quite well with the thin plate theory except in the vicinity of the plate corner. In the proximity of the corner, the Q_θ distributions predicted by Mindlin theory exhibits an abrupt change from a positive value to a negative value. For a very small h/r_o value, the nature of Q_θ increases very rapidly at the corner. The shear force Q_r predictions of both Mindlin and thin plate theories agree reasonably well as shown in Fig. 1b, though significant differences aries near the S'-type edge.

2. FF-plate: The shear force Q_θ distributions due to the two theories agree except in the region near the corner as shown in Fig. 2a. However, the Q_θ distributions by Mindlin theory change rapidly over the narrow zone close to the corner and high Q_θ value arise at the corner as the h/r_o ratio decreases. This trend is very similar to the case of Q_θ for the S'S'-plate. It may be seen from Fig. 2b that the Q_r curves exhibit abrupt changes at distances from the free edge approximately equal to the plate thickness and Q_r reaches its maximum value near the free edge, where it is zero. When the thickness of the plate increases, the maximum value of Q_r predicted by Mindlin theory decreases. As the h/r_o ratio decreases, the maximum value of Q_r tends toward that predicted by thin plate theory.

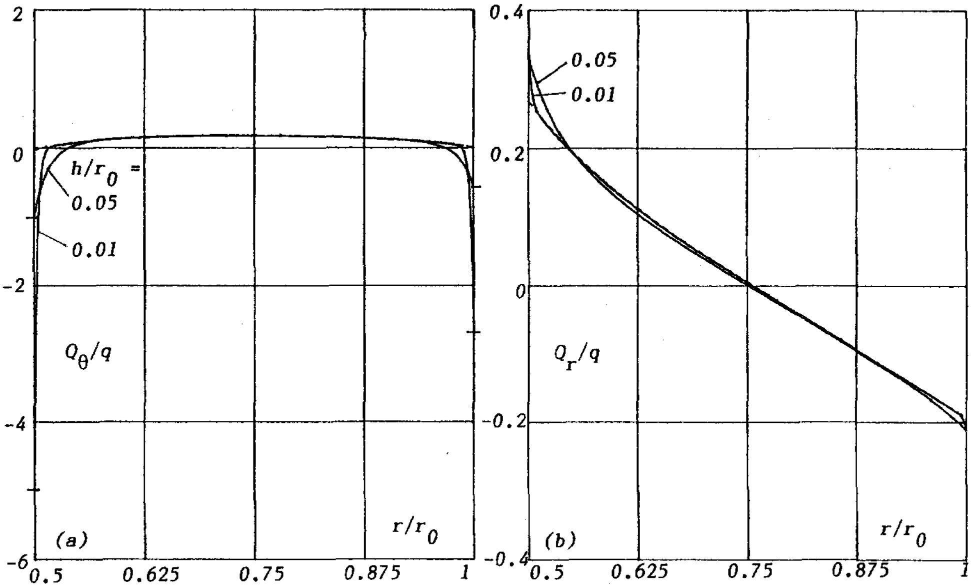

FIG. 1. Variations of shear forces of S'S'-plate: (a) Q_θ along the edge $\theta=0$ and (b) Q_r along the centre line $\theta=30^0$.

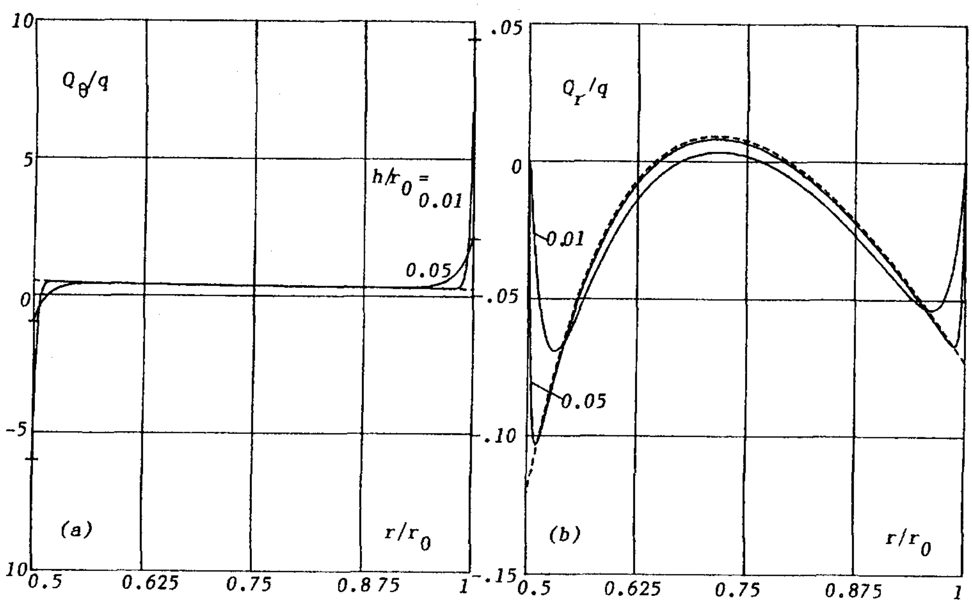

FIG. 2. Variations of shear forces of FF-plate: (a) Q_θ along the edge $\theta=0$ and (b) Q_r along the centre line $\theta=30^0$.

4. CONCLUSIONS

A method of solution has been developed for the bending response of annular sector Mindlin plates. The results can be served as reference to approximate solutions, especially to a 'shear locking' test in the finite element analyses.

REFERENCES [1] K. Marguerre and H.-T. Woernle (1969) Elastic plates, Blaisdell, MA. [2] H. Kobayashi and K. Sonoda (1989) Int. J. Mech. Sci., 31, 679-692.