ANALYSIS OF FLOOD FLOW IN LOW LAND AREA BY TWO DIMENSIONAL MODEL

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1. INTRODUCTION

The low land area in a watershed has been associated to our activity since the human birth and therefore have been cultivated and developed in view of socio-economic, technical, cultural and flood control aspects. In the modelling of extensive inundated plains, one can not simulate the water course as a one dimensional conceptual model. In the present study a two dimensional model is formulated for the simulation of heavy rainfall in Ogura basin. The basin is located in the southern part of Kyoto and also is a junction of two tributaries, Kizu and Katsura to the main river of Uji(Yodo). To protect the area from flooding a pump is installed into the basin whose capacity is decided into the present study. To have the sufficient water at the pump, two cases for a drainage channel were assumed:

Case 1: A channel of width 20m in the middle of the cells whose location is marked in Fig.1 was assumed. The figure also shows the position of two barrier banks in the basin. Case 2: A straight channel of uniform bottom elevation of width 231m (cell northward length) and distance of about 3km from the pump was assumed. The fundamental equations consist of continuity and momentum equations in two space dimensions(x,y). The staggered finite difference scheme was used for the solution.

2. BASIC MATHEMATICAL MODELS AND FINITE DIFFERENCE EQUATIONS OF TWO DIMENSIONAL UNSTEADY FLOW

2.1 TWO DIMENSIONAL RAINFALL SIMULATION MODEL

The plane one layered mathematical models derived from the basic hydrodynamic principles can realize the actual simulation of rainfall in a flood plain, which are(1)

(continuity equation)

$$\frac{\partial h}{\partial t}+\frac{\partial M}{\partial x}+\frac{\partial N}{\partial y}\fallingdotseq r_e \qquad (1)$$

(momentum equation)

x-wise

$$\frac{\partial M}{\partial t}+\frac{\partial}{\partial x}(uM)+\frac{\partial}{\partial y}(vM)=-gh\frac{\partial H}{\partial x}-\frac{\tau_{bx}}{\rho} \qquad (2)$$

y-wise

$$\frac{\partial N}{\partial t}+\frac{\partial}{\partial x}(uN)+\frac{\partial}{\partial y}(vN)=-gh\frac{\partial H}{\partial y}-\frac{\tau_{by}}{\rho} \qquad (3)$$

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where H: water stage, M: the X-wise flow flux(=uh), N: the y-wise flow flux(=vh), x and y : the length elements in the x-and y-direction, re: effective rainfall, h: the water depth, and ss: the shearing stresses on channel bed in the x- and y-direction and are gn

2.2 FINITE DIFFERENCE REPRESENTATION OF EQUATIONS:

Using the staggered difference scheme, the eqs.(1)-(3) are transformed into finite difference form as:

Equation of continuity:

$$\frac{h^{n+3}_{i+1/2,j+1/2}-h^{n+1}_{i+1/2,j+1/2}}{2\Delta t}-\frac{M^{n+2}_{i+1,j+1/2}-M^{n+2}_{i,j+1/2}}{\Delta x}-\frac{N^{n+2}_{i+1/2,j+1}-N^{n+2}_{i+1/2,j}}{\Delta y}=r_e \qquad (4)$$

Equation of x-wise linear momentum equation ignoring non-linear terms:

$$\frac{M^{n+2}_{i,j+1/2}-M^{n}_{i,j+1/2}}{2\Delta t}=-g\frac{(h^{n+1}_{i+1/2,j+1/2}+h^{n+1}_{i-1/2,j+1/2})(H^{n+1}_{i+1/2,j+1/2}-H^{n+1}_{i-1/2,j+1/2})}{2\Delta x}$$
$$-gn^2_{i,j+1/2}\frac{\bar{u}_{i,j+1/2}\sqrt{(u^n_{i,j+1/2})^2+(V^n_{i,j+1/2})^2}}{[(h^{n+1}_{i+1/2,j+1/2}+h^{n+1}_{i-1/2,j+1/2})/2]^{1/3}} \qquad (5)$$

in which $\bar{u}_{i,j+1/2}=(M^{n+2}_{i,j+1/2}+M^{n}_{i,j+1/2})/(h^{n+1}_{i+1/2,j+1/2}+h^{n+1}_{i-1/2,j+1/2})$

Equation of Y-wise linear momenttum can also be expreesed in the same way as eq.(5)

NON-LINEAR TERMS EXPRESSION:

$$\frac{\partial}{\partial x}(uM)=\frac{(uM)^*_{i+1/2}-(uM)^*_{i-1/2}}{\Delta x};\qquad (uM)^*_{i+1/2}=\begin{cases}u^*_{i+1/2}M^* & (u^*_{i+1/2}>0)\\ u^*_{i+1/2}M^*_{i+1} & (u^*_{i+1/2}<0)\end{cases}$$
$$\text{where}\quad u^*_i=\frac{2M^*_i}{h^*_{i+1/2}+h^*_{i-1/2}};\quad u^*_{i+1/2}=\frac{u^*_i+u^*_{i+1}}{2}; \qquad (6)$$

The solution at t=(n+2) Δt was obtained by using the solution at t=n Δt and (n+1) Δt. First M^{n+2} and N^{n+2} was obtained from eq.(5) with eq.(6) and the similar equation of y-wise component and then h^{n+3} was calculated by using eq.(4).

The boundary conditions used in numerical analysis are in the following :

(i) At solid boundary like banking, hillsides and similar ones M=N=0 and h=0.

(ii) At the pumping station for drainage, the pump operation policy is defined and is transformed into the flow flux.

(iii) If the water depth at higher elevation cell is less than ε(say .001m) at the preceding time, then the in-and outflow fluxes at the higher elevation cell are assumed zero.

(iv) The computed flow flux is replaced by 0 at the cell where the estimated depth of water is less than ε.

(v) The negative depth, if computed, is replaced by 0.

(vi) Non-linear terms with respect to temporal scale are estimated by two values at t=(n-2) Δt and t=n Δt which are

$$u^* = (u^{n-2} + u^n)/2 ; \quad M^* = (M^{n-2} + M^n)/2$$

(vii) The time step of computation is Δt =20 sec.

(viii) Effective rainfall is assumed to be uniformly distributed throughout the study area. The rainfall pattern is shown in Fig.2.

4. RESULTS

(i) To verify the model, the continuity equation was checked at every computational time. The results are shown in Table 1 at few computational time.

(ii) To decide the pump capacity, pumps of different capacity were used. And it was found that a pump of capacity 50 m3/s best fit to drain the water from this low land area.

(iii) Hydraulic influence differences of the drainage systems and without drainage system on the overland flow are shown in Fig.3-5.

(iv) The drainage channel considered in case 2 was found to be more effective than case 1. It can be seen from Fig.3-4 and Table 2. It may be because of low conveyance velocity of the area. In case 2, the pump operates at its full capacity after 13 hrs. of rainfall and to drain most of the water from the basin it needs approximately 32 more hours after the rainfall stops. It can be seen in Fig.6.

5. CONCLUSION

Two dimensional modelling is found to be very neccessary for flat land and urban areas where the flow direction is not known. It is also neccessary for the flow simulation in very irregular river and flooded area adjacent to the main channel and tributaries which is the next objective of this study.

All the numerical computations made in the present study have been made by use of the FACOM M-382 at the Data Processing Centre, Kyoto university.

6. ACKNOWLEDGMENT:

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7. REFFERENCES:

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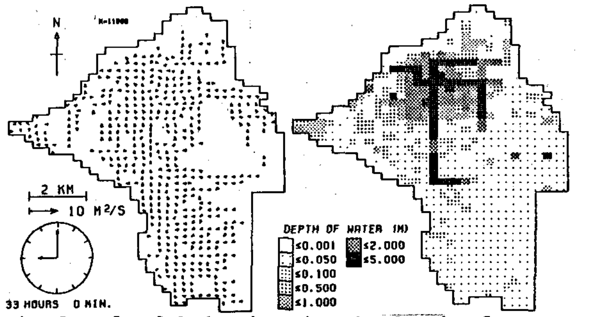

Fig.3 Flood behavior in the basin for case 1 after 22 hrs. of peak rainfall

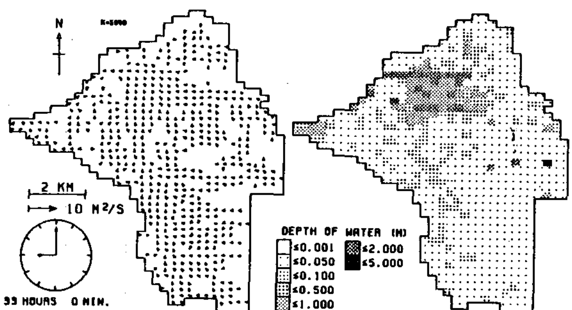

Fig.4 Flood behavior in the basin for case 2 after 22 hrs. of peak rainfall

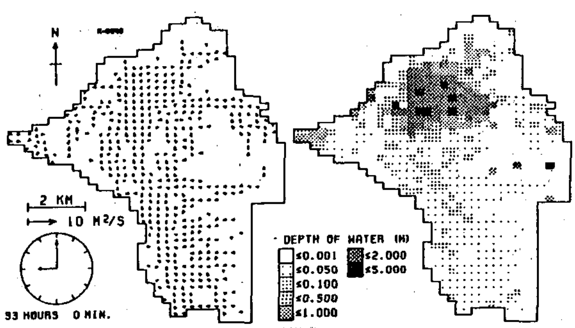

Fig.5 Flood behavior in the basin without drainage (pumping) system after 22 hrs. of peak rainfall

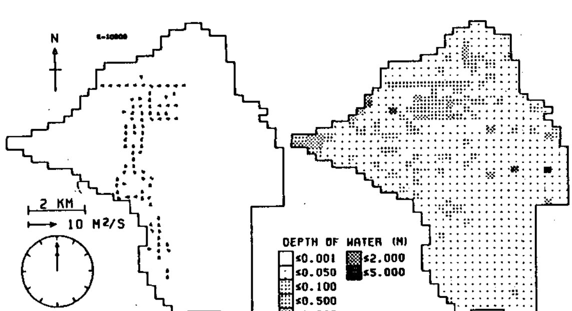

Fig.6 Flood behavior in the basin for case 2 after 28 hrs. of rainfall stop

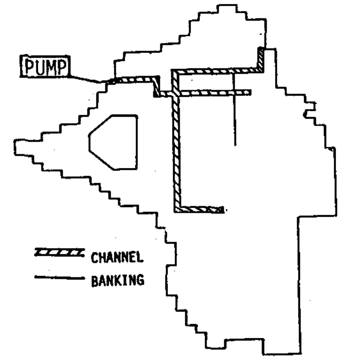

Fig.1 Location of drainage channel & banking in case 1

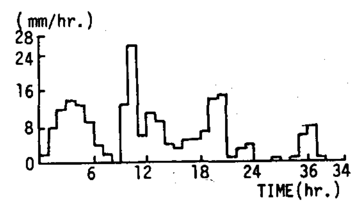

Fig.2 Rainfall pattern

Table 1 Result of continuity check

TIME (hr.)	VCHK CASE1	VCHK CASE2
6	1.00000	1.00032
12	1.00000	1.00040
18	1.00000	1.00068
24	1.00000	1.00102
30	1.00000	1.00141
33	1.00000	1.00156
50	1.00000	1.00214
60	1.00000	1.00224
VCHK=(STORAGE+OUTFLOW)/INFLOW		

Table 2 Computed result of storage & outflow

CASE	AFTER 22 hrs OF PEAK RAINFALL	
	STORAGE(m^3)	OUTFLOW(m^3)
1	10.346×10^6	0.4360×10^6
2	6.571×10^6	4.1947×10^6
WITHOUT DRAINAGE SYSTEM	10.782×10^6	-
CASE2; AFTER 28 hrs. OF RAINFALL STOP STORAGE: 2.8547×10^6 m^3 OUTFLOW: 7.9037×10^6 m^3		