Shakedown Analyses of Two-Span Beams with Consideration of Bending and Shear

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1. Introduction In shakedown analyses, the method of nonlinear programming is generally used. However, in the case of two-span beam under a single repeated moving load, it may be much easier treated as a problem of quadratic programming, for there are only two variables in the mathematical programming.

2. Shakedown Analysis Figure 1 shows the beam model, where i(=1,2,...,n+1) indicates the discrete location of view point. Denoting the elastic bending moments and shearing forces under the repeated moving load by $M_i(\xi)$, $S_i(\xi)$, an applied elastic stress domain in m_i-s_i plane is obtained as follows:

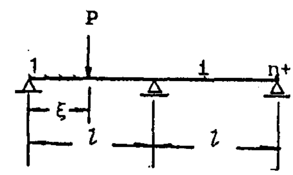

Fig. 1 Two-span beam

$$R_i = \left\{ m_i, s_i \,\middle|\, |m_i| \le \left|\frac{M_i(\xi)}{M_p}\right| , \ |s_i| \le \left|\frac{S_i(\xi)}{S_p}\right| , \ 0 \le \xi \le 2l \right\} \tag{1}$$

surrounded by slashes in Fig. 2 for the sake of demonstration. Here, a circumscribed polyhedron $\bar{R}_i$ about R_i is defined so as to be convex and has some vertexes (m_{ij}, s_{ij}), as marked by dots.

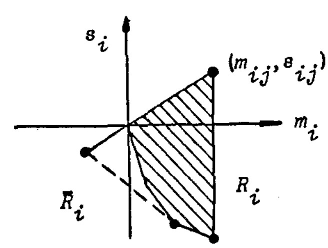

Fig. 2 Vertexes and convex region

Next, a residual reactive force X being assumed at the intermediate support, the residual bending moment and shearing force at each view point are given as $\bar{M}_i, \bar{S}_i$ which are nondimensionlized into $\bar{m}_i = \bar{M}_i/M_p, \bar{s}_i = \bar{S}_i/S_p$. Further, introducing a load multiplier $\lambda = Pl/M_p$ and a nondimensional variable, $\mu = Xl/M_p$, Melan's theorem for determining the shakedown load factor λ_s yields the following mathematical programming:

$$\lambda_s = \text{maximize } \lambda \qquad \text{subject to} \qquad (\lambda m_{ij} + \mu \bar{m}_i)^2 + (\lambda s_{ij} + \mu \bar{s}_i)^2 \le 1 \tag{2}$$

The inequality constraints in problem (2) may be expressed as a quadratic inequality about μ as

$$(\bar{m}_i^2 + \bar{s}_i^2)\mu^2 + 2(m_{ij}\bar{m}_i + s_{ij}\bar{s}_i)\lambda\mu + (m_{ij}^2 + s_{ij}^2)\lambda^2 - 1 \le 0 \tag{3}$$

from which a necessary condition to have a real solution of μ yields

$$\lambda \le \sqrt{\bar{m}_i^2 + \bar{s}_i^2} \Big/ |m_{ij}\bar{s}_i - s_{ij}\bar{m}_i| \tag{4}$$

While inequality (3) gives a constraint for as

$$\gamma_{ij}^- \le \mu \le \gamma_{ij}^+ , \quad \begin{matrix}\gamma_{ij}^- \\ \gamma_{ij}^+\end{matrix} = \frac{1}{\bar{m}_i^2 + \bar{s}_i^2} \left\{ -(m_{ij}\bar{m}_i + s_{ij}\bar{s}_i)\lambda \pm \sqrt{\bar{m}_i^2 + \bar{s}_i^2 - (m_{ij}\bar{s}_i - s_{ij}\bar{m}_i)^2\lambda^2} \right\} \tag{5}$$

Consequently, shakedown load factor λ_s satisfying inequalities (4) and (5) can be solved by applying following expression

$$\mu_s = \underset{i,j}{\text{maximize}}\ \gamma^{-}_{ij} = \underset{i,j}{\text{minimize}}\ \gamma^{+}_{ij} \tag{6}$$

in an iterative procedure within the region

$$0 \leq \lambda_s \leq \lambda_o \quad , \quad \lambda_o = \underset{i,j}{\text{minimize}} \left\{ \sqrt{\bar{m}_i^2 + \bar{s}_i^2} \Big/ |m_{ij}\bar{s}_i - s_{ij}\bar{m}_i| \right\} \tag{7}$$

The graphically demonstrative procedure is shown in Fig. 3, where λ_e and λ_s means elastic limit load factor and shakedown load factor, respectively, while μ_s represents the residual reactive force factor at intermediate support when the beam shakes down.

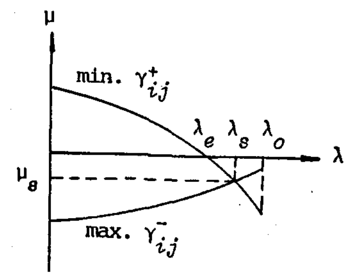

Fig. 3 iterative procedure

3. Results Shakedwon load factor via $M_p/(S_p l)$ is demonstrated drawn in Fig. 4. Four parallel lines mean the ultimate load factor determined by the method of limit analysis, shakedown load factor, elastic limit load factor, residual reactive force factor for consideration of bending alone. Their values are also shown in the figure. It may be seen that shakedown load factor fluctuates significantly with the value of $M_p/(S_p l)$ lying between 0.1 and 1.1, and that discrepency between λ_e and λ_s becomes very little if the value of $M_p/(S_p l)$ greater than about 0.25 while it keeps steady if the value less than about 0.3.

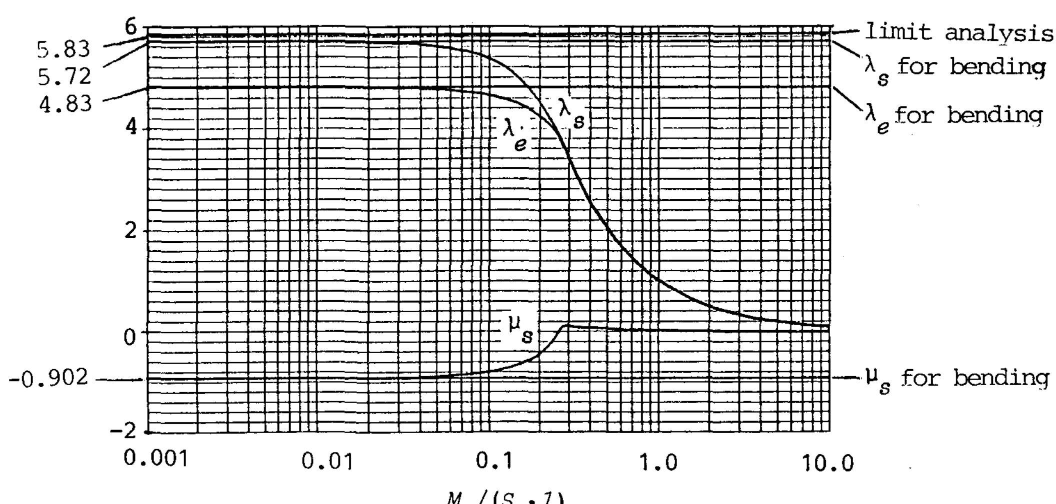

Fig. 4 Variation of $\lambda_s, \lambda_e, \mu_s$ with respect to $M_p/(S_p \cdot l)$

Reference : M.Z. Cohn and G.Maier, Engineering Plasticity by Mathematical Programming, Chap. 6, Pergamon Press.