# NUMERICAL SIMULATION OF UNSTEADY FLOW IN SETA RIVER

by
IWASA, Y., Professor
and
GARG, A. K., Ph.D. Student
Department of Civil Engineering
KYOTO UNIVERSITY

## 1. INTRODUCTION

There is great need for fast and accurate methods for the numerical solution of the equations of unsteady flow. Methods for numerical solution may be classified as: (1) Direct method; and (2) Characteristic method. In Characteristic method the equations are first transformed into the characetristic form before finite difference representations. The finite difference scheme may be classified further into explicit and implicit scheme. In explicit scheme, unknowns can be evaluated a few at a time. In implicit scheme, the finite difference equations are generally non-linear algebraic equations in which the unknowns occure implicitly.

This paper presents a comparision of CPU time and results from three of the more important numerical methods, namely four point implicit method, characteristic method based on explicit scheme and diffusion model, for unsteady flow in Seta river. The implicit scheme used for diffusion model is found to have the advantages of economy in computer time and accuracy and stability under a wide range of time increment.

## 2. NUMERICAL METHODS

Saint-Venant equations for gradually varying flow in open channels are

$$\partial A/\partial t+\partial Q/\partial x=q \tag{1}$$

$$\partial Q/\partial t+\partial (Q^2/A)/\partial x+gA(\partial h/\partial x+S_f)-\frac{qQ}{A}=0 \tag{2}$$

Where A is the flow area, Q the discharge, $S_f$ the friction slope, t the time, h the water surface elevation above a reference horizontal datum, q the lateral inflow, g the acceleration gravity.

### 2.1 FOUR POINT IMPLICIT METHOD

By using the four point implicit method, the above equations are expressed as follows:

$$\frac{1}{2\Delta t}[(A_{i+1}^{n+1}+A_i^{n+1})-(A_{i+1}^n+A_i^n)]+\frac{1}{\Delta x_i}[\theta(Q_{i+1}^{n+1}-Q_i^{n+1})+(1-\theta)(Q_{i+1}^n-Q_i^n)]-q_i=0 \tag{3}$$

$$\frac{1}{2\Delta t}[(Q_{i+1}^{n+1}+Q_i^{n+1})-(Q_{i+1}^n+Q_i^n)]+\frac{1}{\Delta x_i}[\theta\{(Q^2/A)_{i+1}^{n+1}-(Q^2/A)_i^{n+1}\}+(1-\theta)\{(Q^2/A)_{i+1}^n-(Q^2/A)_i^n\}]+\frac{g}{2}[\theta(A_{i+1}^{n+1}+A_i^{n+1})+(1-\theta)(A_{i+1}^n$$

$$+A_i^n)]\{\frac{1}{\Delta x}\{\theta(h_{i+1}^{n+1}-h_i^{n+1})+(1-\theta)(h_{i+1}^n-h_i^n)\}+1/2\{\theta(Sf_{i+1}^{n+1}+Sf_i^{n+1})+(1-\theta)(Sf_{i+1}+Sf_i)\}]-\frac{q_i^n}{2}[\theta\{(Q/A)_{i+1}^{n+1}+(Q/A)_i^{n+1}\}+(1$$

$$-\theta)\{(Q/A)_{i+1}^n+(Q/A)_i^n\}]=0 \tag{4}$$

Where θ is the weighting parameter on which the stability of solution depends. Kanda and Kitada[4] have reported that the solutions are most favorable for θ=0.55~0.60

Two additional equations for determining all the unknowns are supplied by the boundary conditions. The Newton iteration method is used for the solution of this system.

### 2.2 CHARACTERISTIC METHOD

Expressing eq.(2) in the following form :

$$1/g\,\partial v/\partial t+v/g\,\partial v/\partial x+\partial h/\partial x+S_f=0 \tag{5}$$

Introducing the total energy head, $H=h+v^2/2g$, eq.(5) is transformed into

$$1/g\,\partial v/\partial t+\partial H/\partial x=-S_f \tag{6}$$

Eqs. (1)and (6) can be transformed in characteristic form as:

$$dx/dt=v\pm c \tag{7}$$

$$(\partial A/\partial t+\partial Q/\partial x-q)\pm cB(1/g\,\partial v/\partial t+\partial H/\partial x+S_f)=0 \tag{8}$$

Where $c=\sqrt[2]{gA/B}$ and B: the surface width.

Under the assumption of $v^2-c^2<0$ and refering Fig.1, the eqs.(7) and(8) are expressed in explicit form (ref.3) as;

Along $dx/dt=v+c$

$$\{\frac{(A_i^{n+1}-A_i^n)}{\Delta t}+\frac{(Q_i^n-Q_{i-1}^n)}{\Delta x_l}-q_i^n\}+(cB)_i^n\{\frac{1}{g}\frac{(v_i^{n+1}-v_i^n)}{\Delta t}+\frac{(H_i^n-H_{i-1}^n)}{\Delta x_l}+\frac{(Sf_{i-1}+Sf_i^{n+1})}{2}\}=0 \tag{9}$$

Along $dx/dt=v-c$

$$\{\frac{(A_i^{n+1}-A_i^n)}{\Delta t}+\frac{(Q_{i+1}^n-Q_i^n)}{\Delta x_r}-q_i^n\}-(cB)_i^n\{\frac{1}{g}\frac{(v_i^{n+1}-v_i^n)}{\Delta t}+\frac{(H_{i+1}^n-H_i^n)}{\Delta x_r}+\frac{(Sf_i^{n+1}+Sf_{i+1})}{2}\}=0 \tag{10}$$

At new time steps, the solutions at all the interior points can be obtained from simaltaneous eqs.(9) and(10); for the downstream point use eq.(9), and for the upstream point use eq.(10) with boundary conditions.

### 2.3 DIFFUSION METHOD

In this model, the first two inertial terms in eq.(2) are neglected. So eq.(2) becomes

$$S_f=-\partial h/\partial x \tag{11}$$

Yoshiaki IWASA; Arvind Kumar GARG

Accordingly, Manning's equation becomes

$$Q=\frac{1}{n}AR^{2/3}\frac{(-\partial h/\partial x)}{|\partial h/\partial x|^{1/2}} \tag{12}$$

Expressing eq.(1) and eq.(12) in implicit finite difference form

$$\frac{1}{2\Delta t}[(A_{i+1}^{n+1}+A_i^{n+1})-(A_{i+1}^{n}+A_i^{n})]+\frac{1}{\Delta x_i}(Q_{i+1}^{n+1}-Q_i^{n+1})-q_i=0 \tag{13}$$

$$Q_{i+1}^{n+1}=\frac{\{A_{i+1}^{n+1}(R_{i+1}^{n+1})^{2/3}(h_i^{n+1}-h_{i+1}^{n+1})/\Delta x\}}{n_{i+1}|(h_i^{n+1}-h_{i+1}^{n+1})/\Delta x_i|^{1/2}} \tag{14}$$

*Combining eq.(14) with eq.(13) and using the boundary conditions, the obtained system of non-linear equations* are solved by Newton iteration method.

## 3. COMPARISION

For a comparision of three methods, a unsteady flow in Seta river were simulated. The river is irregular in cross-section and is operated by a gate at the downstream end. A constant discharge was also taken for a power plant near to the downstream. The Manning's roughness factor was used 0.025. A constant time increment, $\Delta t$ and variable space increment, $\Delta x$ were used in the numerical computations. The upstream and downstream boundary condition was a specified stage hydrograph shown in Fig.2 and Fig.3. The computations were performed on a FACOM M382 computer system at the Kyoto university.

Fig.2 and Fig.3 also shows the calculated discharge hydrographs by using the three methods. Accuracy and applicability of the method is illustrated in Fig.4 by plotting the stage and discharge hydrograph at Hashimoto (a point 2420m. downstream from upperend, Toriigawa). The execution time required for the computation are given in Table 1. The diffusion method is found to be efficient among the three, since it converges faster and it requires only one half of the number of algebraic equations required by the dynamic model.

## 4. CONCLUSION

The used non-linear diffusion model is found to be nealy accurate and efficient in computer time for simulating unsteady flow in open channels when reversal flow is significant.

Now the authors are testing the same diffusion model for different flow and channel conditions. The same diffusion model will also be applied to open channel networks for comparing the accuracy and computation time with other models.

## 5. ACKNOWLEDGEMENTS

The authors wish to express their gratitudes to associate prof. Mr. K. Inoue and research associates Mr. S. Aya and Mr. A. Tada for their valuable advices and encouragement during this study.

## 6 REFERENCES

1. Akan, A.O. and Yen, B.C., Diffusion wave flood routing in channel networks, ASCE, Vol.107, HY6, June, 1981, PP.719-732
2. Amein, M. and Fang, C.S., Implicit flood routing in natural channels, ASCE, Vol.96, HY12, Dec., 1970
3. Iwasa, Y. and Inoue, K., Numerical simulation of floods by means of various methods, Proc. International symposium on unsteady flow in open channels, IAHR-BHRA, New Castle-upon-Tyne(UK),, 1976, PP.D2-13-24.
4. Kanda, T. and Kitada, T., An Implicit method for unsteady flows with lateral inflows in urban rivers, Proc. of the 17th congress of IHAR, 1977.

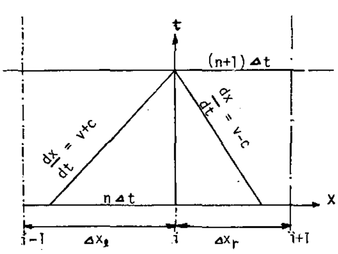


Fig. 1 Characteristic network

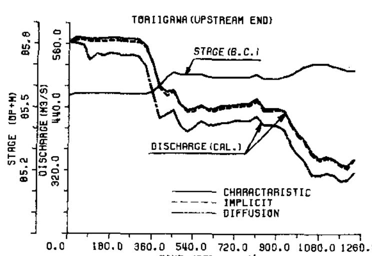


Fig. 2 Hydrographs at upstream

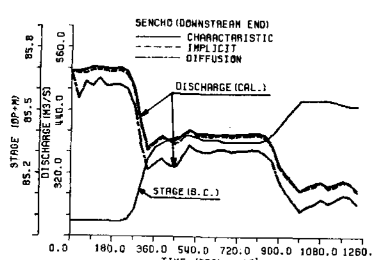


Fig. 3 Hydrographs at downstream

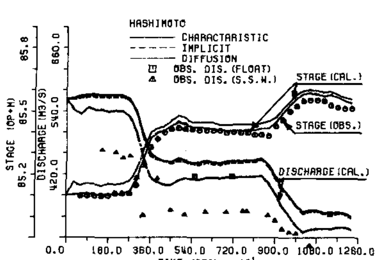


Fig. 4 Hydrographs at Hashimoto

Table 1 : Required Computation Time

| Method | CPU Time (MS) | Time increment (Sec.) |
|---|---|---|
| 1. Implicit | 1127 | 240 |
| 2. Characteristic | 1705 | 10 |
| 3. Diffusion | 1125 | 240 |