A SEMI-DYNAMIC TRAFFIC ASSIGNMENT MODEL AND ITS APPLICATION

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1. Purpose

In most cities, because daily traffic conditions significantly vary, a static traffic assignment model may not sufficiently represent time-varying congestion phenomena in transportation network analysis. On the other hand, a dynamic traffic assignment model requires heavy computational load and does not have a unique solution in most models. Therefore, a semi-dynamic traffic assignment model is an alternative for describing daily traffic dynamics of large-scale networks. This approach formulates static network equilibrium in each period and also considers flow propagation. Flow propagation describes the flow on a link that is unable to exit in a given period and is propagated to the next period. We calculated flow propagation through sensitivity analysis to reduce the computational expenses on the basis of a four-step method. Moreover, we applied the model to the Kanazawa City urban network in order to demonstrate that the semi-dynamic traffic assignment model can be applied to large-scale networks.

2. Overview of logit-based network equilibrium model

Static equilibrium is reached in each period, and a logit-based route choice is assumed as

$$f_{\tau,ij,k} = q_{\tau,ij}\, p_{\tau,ij,k} = q_{\tau,ij} \frac{\exp(-\theta c_{\tau,ij,k})}{\sum_{k \in J_{ij}} \exp(-\theta c_{\tau,ij,k})} \tag{1}$$

where q_{ij} is the demand, $f_{ij,k}$ is the flow on the k-th route, $p_{ij,k}$ is the probability of choosing the k-th route, $c_{ij,k}$ is the travel cost on the k-th route, J_{ij} is the set of routes, θ is a positive parameter, and τ is τ-th period. All of these above variables represent values between the i-th and j-th nodes.

An expression of the above equation is as follows:

$$\mathbf{f}_\tau = \mathbf{Q}_\tau \mathbf{p}_\tau \tag{2}$$

where $\mathbf{f}_\tau$ is the vector of all route flows, and $\mathbf{Q}_\tau$ is the diagonal matrix of travel demands.

The link vector is obtained using route flows and the link-route incidence matrix:

$$\mathbf{x}_\tau = \Delta \mathbf{f}_\tau \tag{3}$$

where $\mathbf{x}_\tau$ (= $x_1, x_2, \ldots, x_{|A|}$) is the vector of all link flows, Δ (= $\{\delta_{a,ijk}\}$)is the link-route incidence matrix, A is the set of links, and $|A|$ is the number of links. In addition, $\delta_{a,ijk}$ is the link-route incidence variable. $\delta_{a,ijk} = 1$ if the k-th route between the i-th and j-th nodes includes the a-th link; otherwise, $\delta_{a,ijk} = 0$. The route travel cost function is given as

$$\mathbf{c}(\mathbf{f}_\tau) = \Delta^T \mathbf{t}(\Delta \mathbf{f}_\tau) \tag{4}$$

where $\mathbf{c}(\mathbf{f})$ is the vector-valued function of route travel cost, and $\mathbf{t}(\Delta\mathbf{f})$ is the vector-valued function of link travel cost. Because the travel cost is a function of its inflow, the probability of route choice is a function of its travel cost. Therefore,

$$\mathbf{f}_\tau = \mathbf{Q}_\tau \mathbf{p}(\mathbf{c}(\mathbf{f}_\tau)) \tag{5}$$

3. Semi-dynamic traffic assignment model and sensitivity analysis method

1) Semi-dynamic traffic assignment model

In this model, all vehicles departing from their origin do not reach their destinations. Some of the vehicles cannot exit the link and are considered as flow propagation. The flow on a link that cannot exit the link in a given period is propagated to the next period; this effect is known as flow propagation. A semi-dynamic traffic assignment model is shown in Figure 1.

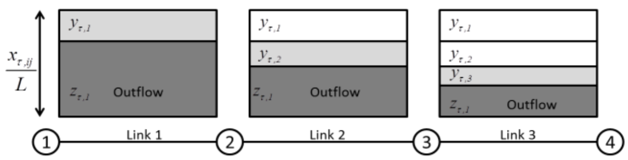

Figure 1. Semi-dynamic traffic assignment model

The inflow into link 1 cannot exit this link and becomes residual flow, which is propagated to the next period. $x_{\tau,a}$ denotes the inflow to the a-th link in the τ-th period; $c_a(x_{\tau,a})$ denotes the travel cost on the a-th link in the τ-th period; $y_{\tau,a}$ denotes the residual flow, which is propagated to the next period on the a-th link in the τ -th period; and z_τ denotes the link flow after the residual flows are eliminated. Thus, the travel time on link 2 should be $c_2(x_{\tau,2}) = c_{\tau,2}(x_{\tau,1} - y_{\tau,1})$, and that on link 3 should be $c_3(x_{\tau,3}) = c_{\tau,3}(x_{\tau,1} - y_{\tau,1} - y_{\tau,2})$. Here $x_{\tau,1}$, $x_{\tau,2}$, and $x_{\tau,3}$ denote the inflow to links 1, 2, and 3, respectively; $y_{\tau,1}$, $y_{\tau,2}$, and $y_{\tau,3}$ denote the residual flow on links 1, 2, and 3, respectively.

If the residual flow is eliminated, not only the travel cost changes but also the inflow changes via network equilibrium. We consider that if the inflow continuously enters a link at the same rate, the travel cost is constant within the same period and that the travel cost is a function of its inflow. On the other hand, the residual flow, which is the function of the travel time on its link, is added on demand between the end node of that link and original destination in the next period.

As previously described, the inflow rate is $x_{\tau,a}/L$. The residual flow on a link in the τ-th period is determined by

$$y_{\tau,ij} = \frac{f_{\tau,ij} t_\tau \delta_{ij}}{L} \tag{6}$$

where $f_{\tau,ij}$ is the flow on a link between the i-th and j-th nodes, t_τ is the link travel cost in the τ-th period, and L is the length of the time period. The summary of the residual flow in the k-th route, where n_{ijk} refers to the number of links in the k-th route between the i-th and j-th nodes, can be written as

$$s_{\tau,ij,n_{ijk}} = \sum_{k'}^{k-1} y_{\tau,ij,n_{ijk'}} \tag{7}$$

Consider that B is the matrix where, $(i,j) = 1$; otherwise, $(i,j) = 0$. Thus, the summary of residual flow is eliminated from route inflow as follows:

$$\mathbf{s}_\tau = \frac{1}{L}\mathbf{T}(\Delta\mathbf{f}_\tau)\sum_{i\in I}\sum_{j\in J_i} f_{\tau,ij}\mathbf{B}_{ij}\boldsymbol{\delta}_{ij} \tag{8}$$

We consider that $\mathbf{r}_{ij} = \mathbf{B}_{ij}\boldsymbol{\delta}_{ij}$, and $\mathbf{R} = (r_{11}, r_{12}, \ldots, r_{|I|\,|Ji|})$; therefore,

$$\mathbf{s}_\tau = \frac{1}{L}\mathbf{T}(\Delta\mathbf{f}_\tau)\mathbf{R}^T\mathbf{f}_\tau \tag{9}$$

The link flow after the elimination of residual flows z_τ is given by

$$\mathbf{z}_\tau = \Delta\mathbf{f}_\tau - \mathbf{s}_\tau \tag{10}$$

From Eq. (5), we have

$$\mathbf{f}_\tau = \mathbf{Q}_\tau\mathbf{p}(\Delta^T\mathbf{t}(\Delta\mathbf{f}_\tau - \mathbf{s}_\tau)) \tag{11}$$

2) Sensitivity analysis

The proposed method with sensitivity analysis in this study includes the following four steps. Step 1: Calculate a static equilibrium in a given time period. Step 2: Design a sensitivity analysis method and obtain flow propagation to the next time period. Step 3: Recalculate a static equilibrium assignment by subtracting the propagated flow from the static equilibrium assignment. Step 4: Deliver the propagated flow to the next time period. The residual flows for flow propagation are approximately calculated using sensitivity analysis to reduce the computational expenses.

On the basis of the sensitivity analysis of the logit-based network equilibrium model, we have

$$\mathbf{f}_s = \mathbf{f}_0 + \nabla_\mathbf{s}\mathbf{f}_0\mathbf{s} \tag{12}$$

where $\mathbf{f}_0$ denotes the flow in a static assignment without residual flow. And the flow $\mathbf{f}$ is given as

$$\mathbf{f} = \Delta\mathbf{f}_0 - \frac{1}{L}\nabla_\mathbf{s}\mathbf{f}_0\mathbf{T}(\Delta\mathbf{f}_0)\mathbf{R}^T\mathbf{f}_0 \tag{13}$$

Thus, with I as a unit matrix, route flows with respect to demand perturbation s are given by:

$$\mathbf{f} = \Delta\mathbf{f_0} + \frac{1}{L}\left[\left(\mathbf{I} - \mathbf{Q}\nabla_\mathbf{c}\mathbf{p_0}\nabla_\mathbf{x}\mathbf{t_0}\Delta\right)^{-1}\mathbf{Q}\nabla_\mathbf{c}\mathbf{p_0}\nabla_\mathbf{x}\mathbf{t_0}\right]\mathbf{T}(\Delta\mathbf{f_0})\mathbf{R}^T\mathbf{f_0} \tag{14}$$

4. Kanazawa City urban network application

The Kanazawa City urban network includes 272 nodes and 964 links. The assumed parameters are θ = 0.5, β = 2, α = 1. Because the time periods of 7 am and 8 am reflect peak hours, the number of vehicles in these time periods is generally greater than those in the other time periods including 6 am and 9 am. Personal trip data was used as the input database. We applied the semi-dynamic traffic assignment model with flow propagation based on the sensitivity analysis to the Kanazawa City urban network in order to obtain the output results of each time period. These results demonstrate that the semi-dynamic traffic assignment model can be applied to a large-scale network.

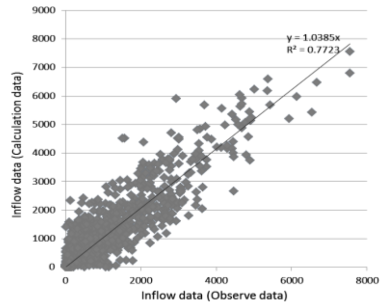

Figure 2. Summary of the inflow data for rush hour periods from 6 – 9 am

5. Conclusion

The sensitivity analysis of a logit-based network equilibrium model was used in a semi-dynamic model. Static network equilibrium was reached in each period, and the dynamics of network flow including flow propagation was considered between each period. By using the results of this analysis, it was determined that the semi-dynamic traffic assignment model can be applied to daily traffic in large-scale networks; However, it is difficult to apply in the proposed method to very large-scale networks because of existing inverse matrix in the sensitivity analysis formulations. The number of path flows is so many that the computer capacity is not enough. Future research is needed to upgrade follow above mention to solve this problem.