SOLVING THE MIXED TRAFFIC ASSINGMENT AS NONLINEAR COMPLEMENTARITY PROBLEM: AN ALTERNATIVE PATH BASED METHOD FOR TRAFFIC USER EQUILIBRIUM ASSINGMENT

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1. Introduction

The interests in mixed traffic flow analysis have been growing up among researchers recently. Unlike developed countries which might face the modal interaction occurring on some urban streets, developing countries, obviously, are experiencing badly on higher level of congestion due to the different traffic compositions as statistically mentioned in KOV (2009), MINH (2005) and so on. Therefore, many researchers from developing countries have begun to work and published some papers related to the mixed traffic. Thus, to follow up with their works related to the mixed traffic flow and because most of the previous works are probably at microscopic analysis, this paper will focus on the network level, that is, the mixed traffic flow user-equilibrium assignment.

2. Literature Reviews

With the existing algorithms either traditional or conventional, the problems of the user-equilibrium traffic assignment, generally known as to find the flow pattern such that no motorists can be better off by unilaterally changing routes have been already solved with some levels of solution accuracies. Zhou, Z, et al (2010) categorizes the algorithms into 3 main groups: link-based, path-based and origin-based algorithms. Due to the computational study of Zhou, Z, et al (2010), there are a plenty of algorithms already applied for the traffic algorithm. Therefore, are there any motivations for other new algorithms? Of course yes, first most of algorithms usually focus on solving convex programming as solving the assignment problems. Second, up to today the most welcomed ones are to the link based and origin based algorithms which usually provide not much information to decision makers. Third, the important one is that almost all of them haven't been experienced with the mixed traffic assignment yet. Owing to these motivations, an alternative path based algorithm will be presented here.

3. Problem formulation

NCP Definition: Let $\Re^n$ be the n-dimensional real linear space and let F be a mapping from $\Re^n$ to $\Re^n$. The complementarity problem consists in finding an $x \in \Re^n_+$, if it exists, such that $F(x) \in \Re^n_+$ and $x^T F(x) = 0$, where $\Re^n_+$ is the nonnegative orthant $\Re^n$, i.e. $\Re^n_+ = \{(x_1,...,x_n) : x_i \ge 0\}$. (Subramanian, 1993)

Multimodal User Equilibrium and NCP

As mentioned in Aashtiani (1979), the minimization of summation of line integrals of link performance function for multiclass user as equivalent to the equilibrium problem is theoretically proposed by Dafermos (1972). Here below is the minimization problem for two-modal assignment.

$$\min z(x_a^1, x_a^2) = \sum_{a\in A} \int_{L:(0,0)}^{(x_a^1, x_a^2)} t_a^1(x_a^1, x_a^2)dx_a^1 + t_a^2(x_a^1, x_a^2)dx_a^2 \quad (1)$$

$$\begin{aligned} s.t \quad & \sum_{k\in K_{rs}^1} f_{k,1}^{rs} - q_{rs}^1 = 0 \quad && \forall r,s \\ & \sum_{l\in K_{rs}^2} f_{l,2}^{rs} - q_{rs}^2 = 0 && \forall r,s \\ & f_{k,1}^{rs} \ge 0;\ f_{l,2}^{rs} \ge 0 && \forall k, \forall l, \forall r,s \end{aligned}$$

where $t_a^1(x_a^1, x_a^2), t_a^2(x_a^1, x_a^2)$: Link performance function of link *a*. x_a^1, x_a^2 : link flow on link *a*. $f_{k,1}^{rs}, f_{l,2}^{rs}$: path flow on path *k* and *l* from an O-D pair *r-s* respectively. q_{rs}^1, q_{rs}^2 : travel demand for an O-D pair *r-s*. K_{rs}^1, K_{rs}^2 : a set of paths for O-D pair *r-s*. Numbers '1' and '2' refer to mode type 1 and mode type 2 respectively.

The demonstration of the equivalence between the two modal user equilibrium assignment and the mathematical minimization program could be found as the same way of that mentioned in Sheffi (1985, pp. 63-65) for the single modal assignment.

The equilibrium conditions are further expressed as the following complementarity problem form:

$$\begin{cases} f_{k,m}^{rs}\left(c_{k,m}^{rs} - u_{rs}^m\right) = 0 & \forall m, \forall k, \forall r,s \\ u_{rs}^m\left(\sum_{k\in K_{rs}^m} f_{k,m}^{rs} - q_{rs}^m\right) = 0 & \forall m, \forall r,s \\ c_{k,m}^{rs} - u_{rs}^m \ge 0 & \forall m, \forall k, \forall r,s \\ \sum_{k\in K_m} f_{k,m}^{rs} - q_{rs}^m \ge 0 & \forall m, \forall r,s \\ f_{k,m}^{rs} \ge 0, u_{rs}^m \ge 0 & \forall m, \forall k, \forall r,s \end{cases} \quad (2)$$

where $c_{k,m}^{rs}$: travel time on path *k* from mode type *m* for the O-D pair *r-s*. u_{rs}^m : minimum travel time for the O-D pair *r-s*.

Suppose $x=\left\{f_{k,m}^{rs},u_{rs}^{m}\right\}^{T}, F(x)=\left\{c_{k,m}^{rs}-u_{rs}^{m}, \sum_{k\in K_m} f_{k,m}^{rs}-q_{rs}^{m}\right\}^{T}$ (3) . Then, $(2)\Rightarrow x^{T}\cdot F(x)=0; x\geq 0; F(x)\geq 0$

4. Methodology

Theorem: Let $F:\Re^{n}\to\Re^{n},\theta$ a strictly increasing function such that $\theta:\Re\to\Re$ with $\theta(0)=0$. Then, z^{*} solves NCP(F) if and only if z^{*} solves the system of nonlinear equations $G_i(z)=0, i=1,2,...,n,$ where $G_i(z)=\theta(|F_i(z)-z_i|)-\theta(F_i(z))-\theta(z_i)$ and $F_i(z)$ is the ***i***th component of $F(z)$. (Mangasarian, 1976)

Subramanian (1993) developed a damped Gauss-Newton algorithm to minimize a function $g(z)=(1/2)\cdot\|G(z)\|^{2}$ in the spirit of least-squares minimization as the method to solve $G_i(z)=0$. In addition $\theta(\alpha)=\alpha|\alpha|$ is used. However, we are interested in Levenberg-Marquardt algorithm instead.

It is obvious that from (3) the sizes of x & $F(x)$ are dependent on the number of paths for each O-D pair of each mode since the vectors x & $F(x)$ are the function of variables: path flows and minimum travel time corresponding to each O-D and to each mode. Therefore, it sounds we should enumerate the paths for each O-D and each mode before solving the NCP. However, a large variety of paths could be enumerated. This has leaded us to apply one of decomposition techniques proposed by Aashtiani (1979) in order to generate the feasible paths with feasible initial solutions.

Computational Exercise

We, here, introduced a studied network, originally from Aashtiani (1979), of two-modal traffic (Mode: type 1 & type 2). Yet, here only considers fix travel demands. The network consists of 7 nodes, 12 one-way links as indicated by the arrows (Fig 1). The mode type 2, or simply called Mode 2 whose routes are marked by dash lines in the network, will share some links together with Mode 1. Each mode has the same O-D pairs. Therefore, the network problem is to find the flow pattern generated from the fix travel demands corresponding to 2 O-D pairs. The inputs for this exercise are: O-D demands $q_{1-7}^{1}=1278; q_{2-7}^{1}=688; q_{1-7}^{2}=1294; q_{2-7}^{2}=4424$, link performance function:
$t_a^m\left(x_a^1,x_a^2\right)=t_a^{0,m}\cdot\left(1+0.15\cdot\left(\frac{x_a^1+0.2\cdot x_a^2}{C_a}\right)^4\right)$ where: $t_a^{0,m}$: Free flow link travel time for mode *m* in min. C_a : Link capacity. x_a^m : Link flow on link *a* by mode *m*. The values of these parameters are:

$t_a^{0,1}=\{5;3;4;6;9;3;4;3;2;7;5;4\};$

$t_a^{0,2}=\{8;6;6;-;12;-;6;-;-;10;7;6\}(-:\text{ no link for mode }2); C_a=\{500;500;500;500;1000;500;1000;500;500;1000;1000;1000\}$

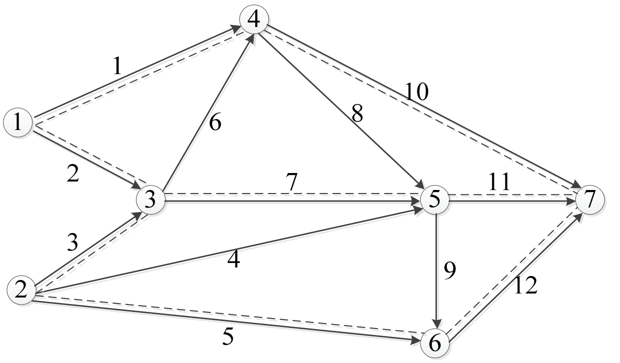

Figure 1: Network Configuration — Mode 1 / Mode 2

Computational results are shown below :

$$\text{Mode 1:}\begin{cases}\text{O-D1}\begin{cases}\text{Path 1: 1-4-7, } f=490\\ \text{Path 2: 1-3-5-7, } f=635, u=16.4703\\ \text{Path 3: 1-3-4-7, } f=152\end{cases}\\ \text{O-D2}\begin{cases}\text{Path 1: 2-5-7, } f=573\\ \text{Path 2: 2-5-6-7, } f=46\ , u=14.7158\\ \text{Path 3: 2-6-7, } f=69\end{cases}\end{cases}$$

$$\text{Mode 2:}\begin{cases}\text{O-D1:}\{\text{Path 1: 1-4-7, } f=1294, u=25.0336\\ \text{O-D2:}\{\text{Path 1: 2-6-7, } f=4420, u=20.3877\end{cases}$$

(Note : the values of path flow above are already rounded.)

5. Conclusion

The algorithm provides more information to the decision maker. Moreover, the higher level of accuracy can be obtained.

Further work

- The algorithm will be applied into the real mixed traffic network.
- To Test performance with the existing algorithms

References

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