A numerical method for multi-crack problems in plane finite brittle solids

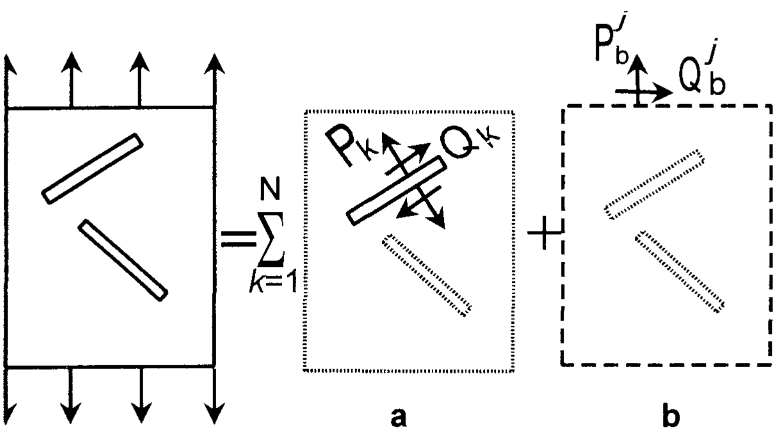
DeFa Wang, Student Member, Graduate School of Fukui University
Takuo Fukui, Member, Department of Architecture and Civil Engineering, Fukui University

1.Introduction:

A numerical method for solving multi-crack interacting problems in plane finite solids is proposed. The basic idea starts from the combination of the "pseudo-traction method" customarily adopted in infinite solids and the BEM customarily used in finite solids. After considering both the traction-free conditions on all crack surfaces and the loading conditions on the outside boundaries, we derive a system of the integral equations, which are evaluated numerically. The stress intensity factors at all crack tips can be obtained with a desirable accuracy. A numerical example shows the high efficiency of this present method.

2.Analysis:

Consider N arbitrary distributed cracks in a finite plate with applied loads acting on the outside boundaries. Using the principle of superposition, the problem is decomposed into N+1 subproblems shown as **Fig.1**.



(1) N single microcrack problems with pseudo-tractions on the crack faces in an infinite plate without remote loads (**Fig.1 (a)**).

(2) A finite plate with no crack while its boundary is divided into M elements at which the fictitious stresses P_b^j and Q_b^j (j=1 to M) are applied (**Fig.1(b)**).

Fig.1 A finite plate with cracks decomposed into (N+1) subproblems.

Considering all the boundary conditions, the following integral equations can be obtained:

$$P_k(t_k) + \sum_{\substack{i=1\\ i\neq k}}^{N} \int_{-a_i}^{a_i} P_i(t_i) f_{nn}(t_i,t_k)dt_i + \sum_{\substack{i=1\\ i\neq k}}^{N} \int_{-a_i}^{a_i} Q_i(t_i) f_{tn}(t_i,t_k)dt_i + \sum_{j=1}^{M} P_b^j g_{nn}(z_j,t_k) + \sum_{j=1}^{M} Q_b^j g_{tn}(z_j,t_k) = 0$$

$$Q_k(t_k) + \sum_{\substack{i=1\\ i\neq k}}^{N} \int_{-a_i}^{a_i} P_i(t_i) f_{nt}(t_i,t_k)dt_i + \sum_{\substack{i=1\\ i\neq k}}^{N} \int_{-a_i}^{a_i} Q_i(t_i) f_{tt}(t_i,t_k)dt_i + \sum_{j=1}^{M} P_b^j g_{nt}(z_j,t_k) + \sum_{j=1}^{M} Q_b^j g_{tt}(z_j,t_k) = 0$$

$$-a_k < t_k < a_k, \qquad 1 \le k \le N$$

$$\sum_{i=1}^{N} \int_{-a_i}^{a_i} P_i(t_i) f_{nn}(t_i,z_j)dt_i + \sum_{i=1}^{N} \int_{-a_i}^{a_i} Q_i(t_i) f_{tn}(t_i,z_j)dt_i + \sum_{m=1}^{M} P_b^m g_{nn}(z_m,z_j) + \sum_{m=1}^{M} Q_b^m g_{tn}(z_m,z_j) = T_n^j$$

$$\sum_{i=1}^{N} \int_{-a_i}^{a_i} P_i(t_i) f_{nt}(t_i,z_j)dt_i + \sum_{i=1}^{N} \int_{-a_i}^{a_i} Q_i(t_i) f_{tt}(t_i,z_j)dt_i + \sum_{m=1}^{M} P_b^m g_{nt}(z_m,z_j) + \sum_{m=1}^{M} Q_b^m g_{tt}(z_m,z_j) = T_s^j \qquad (1)$$

$$1 \le j \le M$$

where $P_k(t_k)$ and $Q_k(t_k)$ are the undetermined pseudo-tractions at the k-th crack while P_b^j and Q_b^j are the undetermined fictitious stresses applied at the mid-point of the j-th element. T_n^j and T_s^j are the loading conditions on the outside boundaries. The influence coefficients f_{nn}, f_{nt}, f_{tt} and f_{tn} are given in [Chen Y.Z.,1984] while g_{nn}, g_{nt}, g_{tn} and g_{tt} are given in [Crouch S.L. and Starfield A.M.,1983] . Once the system is solved, the unknowns will be obtained and the stress intensity factors of the k-th crack can be evaluated by the following formulas:

$$K_I^R = \frac{\sqrt{\pi a_k}}{L}\sum_{l=1}^{L} P_k(a_k \cos\frac{2l-1}{2L}\pi)(1+\cos\frac{2l-1}{2L}\pi), \quad K_{II}^R = \frac{\sqrt{\pi a_k}}{L}\sum_{l=1}^{L} Q_k(a_k \cos\frac{2l-1}{2L}\pi)(1+\cos\frac{2l-1}{2L}\pi)$$

$$K_I^L = \frac{\sqrt{\pi a_k}}{L}\sum_{l=1}^{L} P_k(a_k \cos\frac{2l-1}{2L}\pi)(1-\cos\frac{2l-1}{2L}\pi), \quad K_{II}^L = \frac{\sqrt{\pi a_k}}{L}\sum_{l=1}^{L} Q_k(a_k \cos\frac{2l-1}{2L}\pi)(1-\cos\frac{2l-1}{2L}\pi) \quad (2)$$

Here, a_k is the half-length of the k-th crack, L is the number of the Chebyshev collocation points.

3.Numerical Example:

As shown in **Fig.2**, a homogeneous square finite plate (Al_2O_3) with two collinear crack is considered which is stretched by the tension stress σ_0 in the y direction. Here, assuming that b=1, c=0.5, and the crack inclination angles are equal to zero. $E = 1.792 \times 10^{11}\ N/m^2$, $\nu = 0.207$, $K_0 = \sigma_0\sqrt{\pi a_0}$, a_0 is the average half-length of the two cracks. For different crack lengths, the *SIF's* at the crack tips are calculated by using the presented method. As shown in **Table 1**, the calculated results are compared with those obtained in [Chen Y.Z.,1988] which proves the validity of this presented method in solving multi-crack interaction problem in finite bodies.

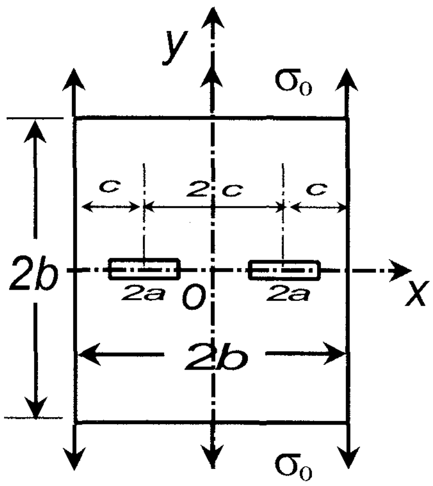


Fig.2 A finite plate with two collinear cracks.

Table 1 The normalized values of the stress intensity factors

2a/b	0.1	0.2	0.3	0.4	0.5	0.6
K_{IR}/K_0	1.0027	1.0184	1.0465	1.0899	1.1543	1.2504
K_{IR}/K_0 [Chen]	1.0054	1.0223	1.0524	1.0989	1.1679	1.2699
K_{IL}/K_0	1.0036	1.0205	1.5001	1.0954	1.1621	1.2613
K_{IL}/K_0 [Chen]	1.0053	1.0215	1.0497	1.0919	1.1521	1.2368

4. Conclusions:

An alternative method for solving the multi-crack interacting problems in finite solids is presented. This method is based on the "pseudo-traction method" for interacting cracks customarily adopted in infinite solids and the indirect boundary element method for the outside boundaries customarily used in finite solids. The stress intensity factors at the crack tips can be obtained with a desirable accuracy. Numerical example shows the high efficiency of the present method in solving multi-crack interaction problem in finite bodies.

References:

[1]. Chen Y. Z., "Multiple crack problems of antiplane elasticity in an infinite body." *Engng. Frac. Mech.*, vol.20, pp. 767-775, 1984.

[2]. Crouch S.L. and Starfield A.M., "Boundary element method in solid mechanics", George Allen & Unwin (publishers) Ltd., 1983.

[3]. Chen Y. Z., "Multiple crack problems for finite plate with arbitrary contour configuration", *Engng. Frac. Mech.*, vol.31, pp. 289-295,1988.