Analysis on the growing cracks in plane brittle solids based on the conservation laws

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1. Introduction:

The study on damaged solids due to microcracks has received considerable attentions in the past several decades since the existence, growth and nucleation of microcracks in solids are of significant importance in both mechanical engineering and civil engineering. Now, it is the commonly recognized opinion that microcracks in brittle materials, e.g., concrete, rocks and ceramics, often control overall deformation and failure mechanisms because the disturbed microcracks in such materials not only lead to macrocrack initiation and final failure, but also induce progressive material damage which could be measured through the decrease of strength, stiffness, toughness, stability and residual life of such materials. On the other hand, within the framework of plane, linear fracture mechanics, the conservation laws (*J*-integral, *L*-integral and *M*-integral) were proposed and extremely attractive. The purpose of the present work is to analyze what role the *M*-integral plays in brittle solids with growing microcracks. The basic idea starts from the *M*-integral analysis customarily used in single crack problems, which was regarded as a more natural description of energy release rate for plane cracks, rather than the *J*-integral. As an initial attempt, only the homogenous, plane, brittle solid is considered in this paper.

2. Analysis:

The *M*-integral: The definition of the *M*-integral is well known as (Budiansky and Rice, 1973):

$$M = \oint_c (w \cdot x_i \cdot n_i - T_l \cdot u_{l,i} x_i) ds \tag{1}$$

where w is the strain energy density, T_l is the traction acting on the outer-side of a closed contour C, $U_{l,i} = \partial u_l / \partial x_i$ $(l=1,2,\ i=1,2)$. Assume that the closed contour C encloses all microcracks completely in a two-dimensional brittle solid, and a global coordinate system (x_1, x_2) and a local coordinate system $(x_1^{(k)}, x_2^{(k)})$ are introduced in **Fig. 1**, respectively. In the global system, the *M*-integral could be evaluated as follows:

$$M = \sum_{k=1}^{N} M^{(k)}(x_1, x_2) = \sum_{k=1}^{N} \{\oint_{c^k} (w \cdot x_i \cdot n_i - T_l \cdot u_{l,i} \cdot x_i) ds = \sum_{k=1}^{N} M^{(k)}(x_1^{(k)}, x_2^{k}) + \sum_{k=1}^{N} \{\xi_1^{(k)} J_1^{(k)} + \xi_2^{(k)} J_2^{(k)}\} \tag{2}$$

where $M^{(k)}(x_1^{(k)}, x_2^{(k)})$ is considered in the local system $(x_1^{(k)}, x_2^{(k)})$. In this paper, the calculated values of the *M*-integrals are normalized by $M_0 = \frac{\kappa+1}{8\mu} \cdot \pi \cdot (\sigma_0 \cdot a_0)^2$ where a_0 refers to the average half length of all the microcracks without increment Δa.

Fracture criterion: One of the mixed-mode fracture criterion in brittle solids is given as:

$$(K_I / K_{IC})^2 + (K_{II} / K_{IIC})^2 \geq 1 \tag{3}$$

where K_{IC} and K_{IIC}, respectively, are the critical mode I and mode II stress intensity factors of the weak plane where pre-existing microcracks are located.

3. Numerical Example and Discussions:

Four cases of different microcrack arrangements are considered in this section. Consider two finite microcracks in an infinite plane brittle solid shown in **Fig.2**. The material Al_2O_3 is considered and its critical stress intensity factors are $K_{IC} = 0.259\, MN/m^{3/2}$ and $K_{IIC} = 0.518\, MN/m^{3/2}$, respectively. The remote loads are $\sigma_{xx}^{\infty} = \sigma_{yy}^{\infty} = \sigma_0$. By changing the value of the remote loads, the left microcrack in **Fig.2** firstly begins to grow when the condition (3) is satisfied

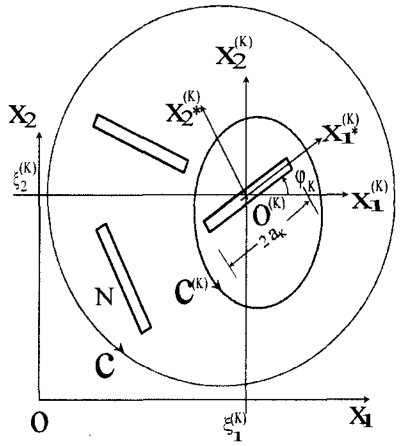

Fig.1 Interacting microcracks in a two-dimensional brittle solid and the closed contours.

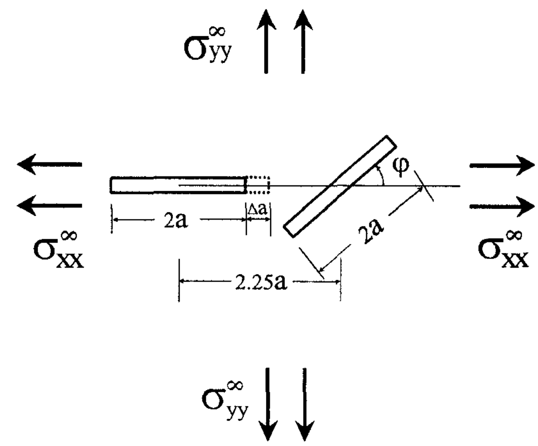

Fig.2 Two interacting cracks under the remote loads.

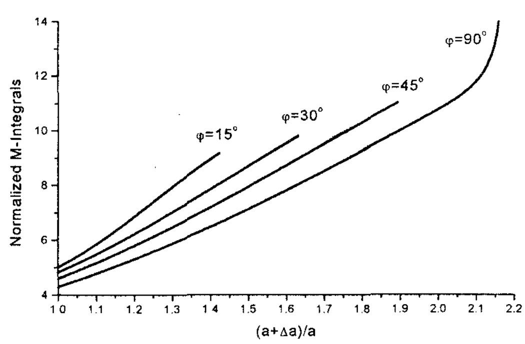

Fig.3 Normalized *M*-integral versus the crack increment.

Then, as shown in **Fig. 3**, the normalized value of the *M*-integral against the increment of the microcrack length can be obtained. Of the most interest is the relation between the values of *M*-integral and the effective elastic moduli for the microcracking solid. From **Fig.3**, we can find that, the value of the *M*-integral will increase gradually corresponding to the increasing increment of the growing microcrack. During the microcrack growth, it is found that the maximum value of the *M*-integral is just corresponding to the microcrack growth at which the largest reduction of the effective elastic moduli occurs. For example, to the case of $\varphi = 30°$, the crack growth $(a+\Delta a)/a = 1.63$ at which the maximum value of the integral occurs coincides well with the largest loss of the effective moduli (the crack's growing is stopped at the growth $(a+\Delta a)/a = 1.63$ by the strong mutual shielding effects among the two cracks even under the constant loads). Quite contrary, the minimum value of the *M*-integral is just corresponding to the microcrack growth at which the smallest reduction of the effective elastic moduli occurs (in this case, the minimum *M*-integral value is corresponding to $(a+\Delta a)/a = 1.0$).

4. Conclusions:

From the physical point of view, it is concluded that an inherent relation does exist between the *M*-integral and the effective elastic moduli for a microcracking brittle solid. In other words, the effective elastic properties could be reformulated by the *M*-integral, although the detailed mathematical formulation between the moduli and the *M*-integral remains to be adequately investigated. It does provide an evidence and a possibility, i.e., there exists an alternative evaluation of the evolutionary damage for microcracking brittle solids based on the *M*-integral analysis. Potential applications of the new description based on the *M*-integral analysis are remarkable. For example, the increasing values of the *M*-integral during an evolutionary microcrack damage just represent the progressive energy release. This topic will inevitably require further investigation and will be discussed in the authors' separate paper.

References:

Budiansky, B. and Rice, J. R., "Conservation laws and energy-release rates," ASME *Journal of Applied Mechanics*, Vol.40, pp.201-203,1973.