

## TRANSMISSION OF LONG-PERIOD WAVES THROUGH THE RUBBLE BASE OF COMPOSITE BREAKWATERS

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**INTRODUCTION:** It is well documented that when the short waves are scattered or diffracted, free long waves can be further generated and propagated away from the scatters at the speed of wave celerity. Breakwaters, often consisting of randomly placed stones, are used to protect harbour and coasts. When a group of short waves is obstructed by porous breakwater, substantial change in the wave pattern occurs due to the combined effects of diffraction, reflection and transmission, and free long-period waves can be generated. An understanding of generation of the long-period waves due to the porous breakwaters, and their reflection and transmission characteristics are then of primary importance. Several studies have been done to estimate the reflection and transmission coefficients of monochromatic short waves for rubble mound breakwaters. The generation of long waves due to the diffraction of short-wave groups by rubble mound breakwaters and their reflection and transmission characteristics remain unattempted. In this paper, we investigate the behavior of long-period waves around the composite-type breakwater due to the reflection and transmission of short-wave groups.

**THEORETICAL FORMULATIONS:** We consider a continuous breakwater having a homogeneous, isotropic vertical rubble base of width  $b$  and height  $a$ , crowned by an impervious element of same width as shown in Fig.1. The water depth is assumed constant at  $h$ . Characteristics of the porous rubble base are expressed by its porosity  $\epsilon$ , linear friction factor  $f$ , and the inertial term  $s$  (Sollitt & Cross 1972).

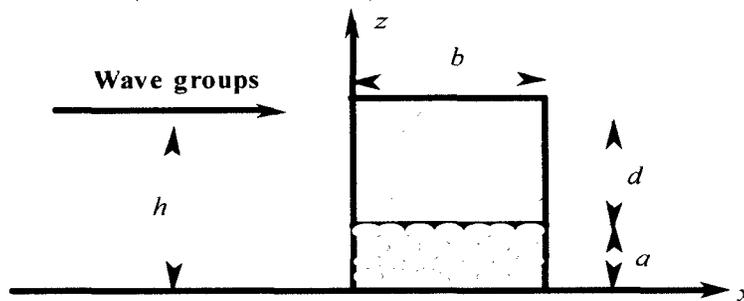


Fig.1 Definition sketch

We assumed that the excitations are provided by a group of two sinusoidal short-wave components with slightly different frequencies. The central frequency and wave number of incident short waves are denoted by  $\omega$  and  $k$  respectively. For an inviscid and incompressible fluid and for irrotational flow, the wave field can be represented by velocity potential  $\Phi$ . By introducing the slow variables  $(x_1, z_1, t_1) = (\epsilon x, \epsilon z, \epsilon t)$ , we write velocity potential in terms of harmonics with respect to the fast time, i.e.

$$\Phi = \sum_{n=1}^{\infty} \epsilon^n \sum_{m=-n}^n \Phi_{nm}(x, z, x_1, z_1, t_1) e^{-im\omega t} \tag{1}$$

with  $\Phi_{n,m}$  being the complex conjugate of  $\Phi_{n,-m}$ . In this solution form,  $\Phi_{11}$  represents the short-wave potential at the leading order, and  $\Phi_{10}$  is the long-wave potential. Because the incident wave group is the superposition of two sinusoidal waves of slightly different frequencies, the short-wave potential  $\Phi_{11}$ , is the sum of the diffraction solution of each wave components. This implies that the existing solution for monochromatic waves can be adopted to compute the reflection and transmission coefficients for short wave components of the wave group. Several different methods can be used to find the reflection and transmission coefficients of short waves. In this paper, we used the methods of eigenfunction expansions presented by Losada et al. (1993). Locked long waves are generated by the self-product of propagating short-wave components and the evanescent modes of the short waves do not contribute to the generation of long waves. Velocity potentials for locked long waves and free long waves can be found from the governing equations of long waves induced by a wave group in water of constant depth (Liu et al. 1991). The matching conditions at the transitions ( $x_1=0$  and  $x_1=b$ ) are specified in terms of the pressure and mass flux, and ensure the continuity of the solutions in the adjacent regions. We assumed that the width of the breakwater  $b$  is small in comparison with the wavelength of wave groups, and the leading order long-wave solution for  $\Phi_{10}$ , which is correct up to  $O(\epsilon)$ , is the same as if the breakwater were absent. Under these assumptions and fulfilling the matching conditions, the following equations for free long waves are derived:

$$E^R = A_{10}(T_1 T_2^* - R_1 R_2^* - 1) + E^T \tag{2}$$

$$E^T = \frac{A_{10} \sqrt{gh}}{2\Omega} \left[ \left( K + \frac{\Omega}{\sqrt{gh}} \right) (1 - T_1 T_2^*) + \left( \frac{\Omega}{\sqrt{gh}} - K \right) R_1 R_2^* \right] \tag{3}$$

where  $E^R$  is the amplitude of reflected free wave,  $E^T$  is the amplitude of transmitted free wave,  $R_1$  and  $R_2$  are the reflection coefficients of two short wave components in the group,  $T_1$  and  $T_2$  are the transmission coefficients of two short wave components in the group,  $K$  and  $\Omega$  are respectively the wave number and frequency of the wave envelop, and  $A_{10}$  is the amplitude of incident locked long wave.

**RESULTS AND DISCUSSION:** Computations are conducted for the geometric parameters  $d/h=0.75$ ,  $b/h=0.5$ . Porosity  $\varepsilon=0.45$ , and the inertial coefficient  $s=1.0$  are assumed for the porous rubble base. Reflection and transmission coefficients of the individual short wave components are computed using eigenfunction expansion (Losada et al. 1993), and the results for the linear friction factor  $f=0.5$  and  $f=1.0$  are shown in Fig.2.

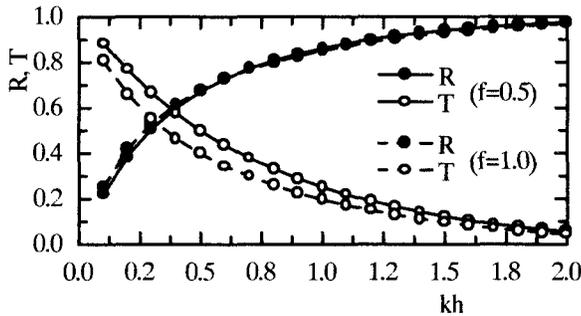


Fig.2 Variation of R and T with  $kh$  and  $f$

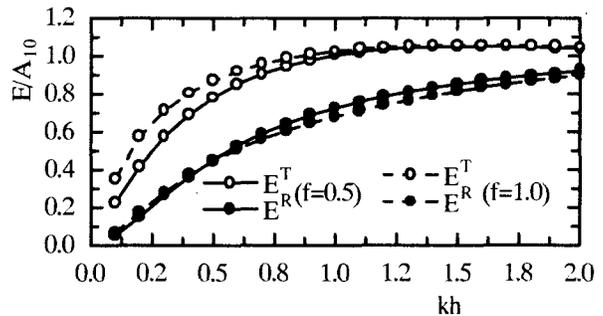


Fig.3 Normalized amplitude of free long waves

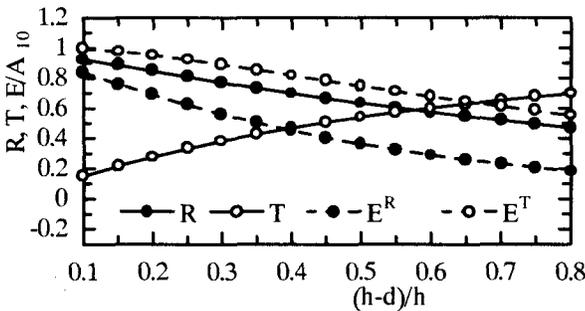


Fig.4 Variation of  $R$ ,  $T$  and normalized  $E$  with the height of porous layer for  $kh=0.8$

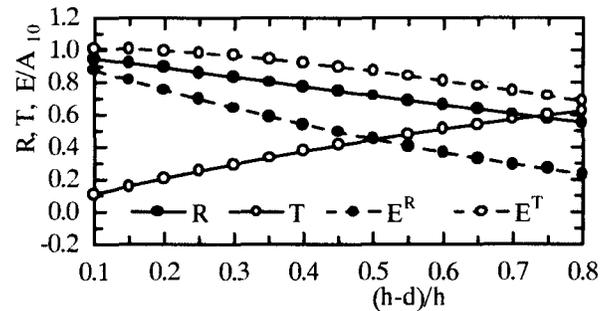


Fig.5 Variation of  $R$ ,  $T$  and normalized  $E$  with the height of porous layer for  $kh=1.0$

In Fig.3, the normalized amplitudes of reflected and transmitted free long waves are presented for different values of  $kh$ . As indicated in Fig.2, shorter waves experience stronger reflection and the longer waves experience stronger transmission. Theoretical result shows that due to the reflection and transmission of wave group by the porous breakwater, free long waves of significant height are generated, and the transmitted free long wave amplitude is larger than the reflected free long wave. Larger transmissions of free long waves are found for shorter short wave components comprising the wave groups. The reflection and transmission of the free long waves are not strongly tied to the damping factor of the porous medium, particularly for the larger values of  $kh$ . However, the amplitude of free long waves increases with the increase in modulation of short waves of the group. Variation of reflection and transmission coefficients and normalized amplitudes of free long waves with the height of rubble base of breakwater are shown in Fig.4 & 5 for  $kh=0.8$  and  $kh=1.0$  respectively. The result indicates that both the amplitudes of reflected and transmitted free long waves decrease with the increase of rubble base height.

**CONCLUSIONS:** Analytical results are obtained for the scattering of short-wave groups by a continuous vertical porous breakwater. It is shown that second-order free long-period waves are generated because of the discontinuity in the locked long waves. The transmitted free long waves reach the maximum amplitude when the reflection of short waves is maximum. It implies that though the conventional rubble mound breakwaters can reflect short waves efficiently, significant amount of free long-wave energy is transmitted to the leeward side.

**REFERENCES:**

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