Modeling of Reinforcement as Beam Element

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1. Introduction

Research for the analysis of RC structures are being carried out by using different methods. Various phenomenon that are occurring in various stages of degradation of the structure, due to various type of loading histories(e.g. monotonic-cyclic, static-dynamic) and structural characteristics (reinforcement distribution, shape of structure), leading finally to the final failure stage is being studied by various researchers.

In this paper, the phenomenon of the effect of bending of reinforcement in compression inside the RC members is presented. Formulation of a new type of beam element for the reinforcement to simulate of such phenomenon is presented. This element is still in developmental stage.

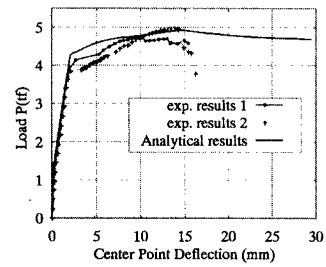
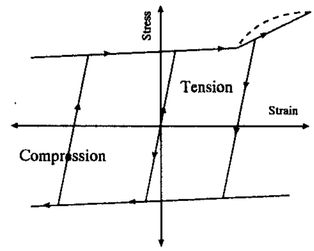


Fig. 1: Typical Exp. and Anal. Results

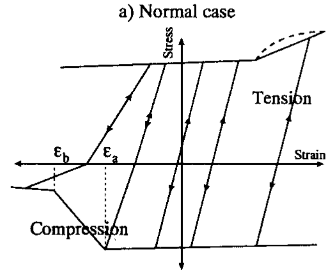
2. The Effect of Bending of Reinforcement

The effect due to bending of reinforcement is noticed in various types of structures. In case of column under axial load, lateral reinforcement bends and provides the required confinement in concrete and was studied experimentally and analytically by Pallewatta et. al.[1] In case of columns under combined axial and cyclic lateral load, the axial reinforcement bends and the effective axial strength decreases considerably in comparison to the uniaxial stress-strain relation(Fig. 1).

In case the reinforcement is modeled as a rod element or a distributed reinforcement, the stress-strain relation then becomes a direct relation between stress and strain. Typical stress-strain relation of the bare reinforcement under uniaxial strain cycles as shown in Fig. 2a. In such models or models taking care of Bauschinger effect (hardening of steel, and tension stiffening effect are characteristics of the reinforcement in tension), reinforcement in compression continues to take load. In a normal case of a column failing in bending mode with reinforcement evenly distributed in the section, the load-displacement behavior predicted by any analytical method will show flat or ductile behavior in the final stage representing the load carried by the reinforcements, even if one assumes that the whole concrete area has failed.



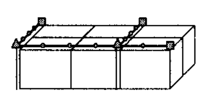
a) Normal case



b) Effect of bending of reinforcement

Fig. 2: Stress-strain of Reinforcement

Some experimental results show considerable ductility, where as others (specially experiments under cyclic loads) show considerable decrease in the strength after some stage. This effect cannot be explained without taking care of the phenomenon of bending of reinforcement. When an approximate stress-strain relation as shown in Fig. 2b is implemented, the decrease in strength, and other characteristics of the unloading envelop can be matched. The prediction of the values of ε_a and ε_b and the characteristics of the unloading cycles cannot be not be done without have a proper understanding of the actual situation that is causing the bending of the reinforcement, namely the bulging effect of concrete due to confinement effect. It is probably quite different from the pure buckling problem of reinforcement that can be simulated by using large displacement theory[2].



▲ Free Rotation
▣ Rotation Disabled

Fig. 3 Modelling of reinforcement and the concrete

3. Formulation of Three Nodded Beam Element

To take care of this problem, the authors are developing a new beam element, where the stress-strain is implemented at gauss point. The connectivity of the beam element and the cubic concrete element is a problem. A simplistic possible solution was proposed as shown in Fig. 3 (Gupta and Tanabe[3] for further details).

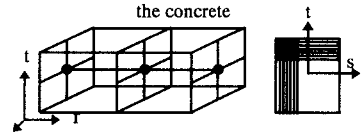


Fig. 4 3 nodded beam element

The beam element is assumed to have three nodes(Fig. 4). The relation between global and member axis coordinate system, the relation between the member axis displacement variables and global nodal variables, and the equations for partial derivatives are shown below, where **T** is transformation matrix

$$(x_m \quad y_m \quad z_m)_r = [\mathbf{T}](x \quad y \quad z)_r = \sum_{i=1}^{3}[\mathbf{T}]N_i(r)(x_i \quad y_i \quad z_i) \tag{1}$$

$$\{u_m \quad v_m \quad w_m \quad \theta_{1m} \quad \theta_{2m} \quad \theta_{3m}\}_r^T = \sum_{i=1}^{3}\begin{bmatrix}\mathbf{T} & 0\\ 0 & \mathbf{T}\end{bmatrix}N_i(r)\{u_i \quad v_i \quad w_i \quad \theta_{1i} \quad \theta_{2i} \quad \theta_{3i}\}^T \tag{2}$$

$$\frac{\partial}{\partial x_m} = J^{*}\frac{\partial}{\partial r} = \left[\partial x/\partial r\right]^{-1}\frac{\partial}{\partial r} = \left[\sum_{i=1}^{3}\left(\partial N_i(r)/\partial r\right)x_i\right]^{-1} \tag{3}$$

The sectional properties are considered at the gauss points by dividing the section into small parts. The displacements and strains are assume to be small. The strain along the axis as a function of r can be written as

$$\begin{Bmatrix}\varepsilon_m\\ \gamma_{ym}\\ \gamma_{zm}\\ \phi_{xm}\\ \phi_{ym}\\ \phi_{zm}\end{Bmatrix} = \begin{Bmatrix}u_{xm}\\ v_{xm}+\theta_{2m}\\ w_{xm}+\theta_{3m}\\ \theta_{1m,xm}\\ \theta_{2m,xm}\\ \theta_{3m,xm}\end{Bmatrix} = \sum_{i=1}^{3}\begin{bmatrix}\mathbf{T} & 0\\ 0 & \mathbf{T}\end{bmatrix}\begin{bmatrix}J^{*}\partial N_i/\partial r & 0 & 0 & 0 & 0 & 0\\ 0 & J^{*}\partial N_i/\partial r & 0 & 0 & N_i & 0\\ 0 & 0 & J^{*}\partial N_i/\partial r & 0 & 0 & N_i\\ 0 & 0 & 0 & J^{*}\partial N_i/\partial r & 0 & 0\\ 0 & 0 & 0 & 0 & J^{*}\partial N_i/\partial r & 0\\ 0 & 0 & 0 & 0 & 0 & J^{*}\partial N_i/\partial r\end{bmatrix}\begin{Bmatrix}u_i\\ v_i\\ w_i\\ \theta_{1i}\\ \theta_{2i}\\ \theta_{3i}\end{Bmatrix} = \mathbf{Bu} \tag{4}$$

The axial stress-strain at each part(s,t) of the section at the gauss point(r) can be calculated as

$$\varepsilon_{xm}(r,s,t) = \varepsilon_m + (h_s/2-s)\phi_{ym} + (h_t/2-t)\phi_{zm} \quad \& \quad \sigma_{xm}(r,s,t) = f\left(\varepsilon_{xm}(r,s,t)\right) = f\left(r,s,t\right) \tag{5}$$

The stress tensor is assumed to consist of

$$\sigma(r,s,t) = \left\{N_{xm}\ \ V_{ym}\ \ V_{zm}\ \ M_{xm}\ \ M_{ym}\ \ M_{zm}\right\}^T = f(\varepsilon_m, \gamma_{ym}, \gamma_{zm}, \phi_{xm}, \phi_{ym}, \phi_{zm}) \tag{6}$$

$$N_{xm} = \int_A \sigma_{xm} dA,\ V_{ym} = \alpha G_s \gamma_{ym} A,\ V_t = \alpha G_t \gamma_{zm} A,\ M_{xm} = K'\phi_{xm},\ M_{ym} = \int_A \sigma_{xm}(h_s/2-s)dA \ \&\ M_{zm} = \int_A \sigma_{xm}(h_t/2-t)dA \tag{7}$$

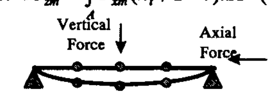


Fig. 5 Beam Element Formulation

The incremental form of Eq. 6-7 can be written as

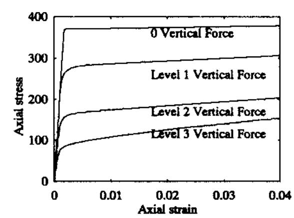
$$d\sigma = d\begin{Bmatrix}N_{xm}\\ V_{ym}\\ V_{zm}\\ M_{xm}\\ M_{ym}\\ M_{zm}\end{Bmatrix} = \begin{bmatrix}\partial N_r/\partial\varepsilon_{xm} & 0 & 0 & 0 & \partial N_r/\partial\phi_s & \partial N_r/\partial\phi_t\\ 0 & \alpha G_s A & 0 & 0 & 0 & 0\\ 0 & 0 & \alpha G_t A & 0 & 0 & 0\\ 0 & 0 & 0 & K' & 0 & 0\\ \partial M_s/\partial\varepsilon_r & 0 & 0 & 0 & \partial M_s/\partial\phi_s & \partial M_s/\partial\phi_t\\ \partial M_t/\partial\varepsilon_r & & 0 & 0 & \partial M_t/\partial\phi_s & \partial M_t/\partial\phi_t\end{bmatrix} d\begin{Bmatrix}\varepsilon_m\\ \gamma_{ym}\\ \gamma_{zm}\\ \phi_{xm}\\ \phi_{ym}\\ \phi_{zm}\end{Bmatrix} = Dd\varepsilon \tag{8}$$

where all the term can easily be calculated. One such examples is given below:

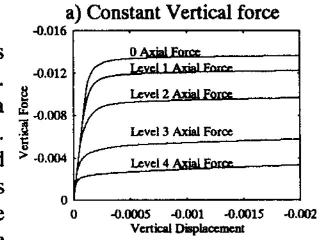
$$\frac{\partial N(\varepsilon_x,\phi)}{\partial\varepsilon_x} = \sum_{j=1}^{N}\frac{\partial\sigma_j}{\partial\varepsilon}\frac{\partial\varepsilon}{\partial\varepsilon_x}dA_j = \sum_{j=1}^{N}\frac{\partial\sigma_j}{\partial\varepsilon}dA_j \tag{9}$$

The stiffness matrix and the nodal force vectors global axis is given as follows

$$K_m = \int_l B^T DB\,dl \qquad\qquad F_m = \int_l B^T\sigma\,dl \tag{10}$$



a) Constant Vertical force



b) Constant Axial force

Fig. 6 Parametric Study of the beam element behavior using two elements

To verify the validity of this formulation, the reinforcement between stirrups is modelled as a simply supported beam with two 3 nodded beam element(Fig. 5). Fig. 6a shows the axial stress-strain behavior when vertical force is first applied to a particular level and then axial load is applied keeping the vertical load constant. Fig. 6b shows the behavior when axial force is applied to a particular level and vertical load is applied keeping the axial load constant. When the beam element is pulled without central vertical force, it resembled the stress-strain curve. In these calculations, increase of application of vertical force level at the center decrease the capacity of or the axial force and vice versa. These results looks logical.

4. Conclusions

Though researchers are doing research of various phenomena to understand the behavior of RC members, not enough work has been done to understand the effect of decrease of strength of reinforcement, specially in case of compression. In this paper, formulation for a new type beam element is proposed for simulation of such phenomena. The initial results looks promising. However, when similar calculation is carried out for axial tensile forces, the strength also decreases. This is not logical and his problem can be overcome by implementation of large displacement theory. The development and implementation of large displacement theory is in process and will be presented in future.

References

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[2] Nakamura H. and Higai T. "Analytical Study on the Plastic Buckling Behavior of Longitudinal Reinforcement", Proc. of the 52nd Annual Conf. of JSCE, V, pp.540-541,1997.

[3] Gupta, S. and Tanabe, T. "3d Analysis using Unified Concrete Plasticity Model with Reinforcement under Compression Modelled as Beam Element", FRAMCOS-3, Gifu, Japan, pp. 1005-1014, Oct., 1998