Description of the Cross-sectional Behavior of Steel Beam Columns by Force-space Plasticity

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1. Introduction

Nearly all the reported force-space plasticity formulations used in the inelastic analysis of steel structures are developed by borrowing the concepts of stress-space plasticity to describe the interaction of the stress resultants on the cross section. However, fiber analysis with the modified two-surface model for structural steel with yield plateau (heretofore referred to as 2SM-SS) reveals that there are certain features of cross-sectional behavior that can not be represented simply by the concepts analogous to stress-space plasticity. The current development of such a force-space plasticity model tries to address these special features in its formulation.

2. Formulation of Force-space Plasticity

The current force-space formulation is based on 2SM-SS, and is calibrated against the fiber analysis results using 2SM-SS. The monotonic stress-strain relation according to 2SM-SS is illustrated in Fig.1. The whole curve can be divided into three stages: the elastic stage (O-A), the yield plateau stage (A-B) and the hardening stage (beyond B).

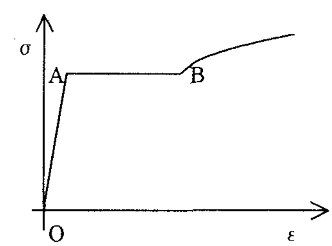

Fig.1 Uniaxial $\sigma-\varepsilon$ curve (2SM-SS)

The force-space model is formulated in a normalized axial force-uniaxial bending moment space (the n-m space), and the forces are scaled by their yield values respectively. The strength curves used are illustrated in Fig.2. They are the yield curve (Y.C), the yield plateau curve (Y.P) and the bounding curve (B.C). The yield curve marks the locus where the initial elastic state ceases. The yield plateau curve represents the force state where most of the fibers of the cross section are in yield plateau stage. And the bounding curve is where the plastic stiffness of the whole section reaches a limit value. Both the location and the size of the bounding curve change during plastic deformation.

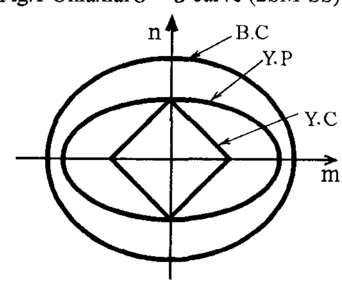

Fig.2 Strength Curves

One of the cross-sectional behavior that should be taken into account is the degeneration of the elastic region. Before any plastic deformations occur, moving of loading point within the yield curve only triggers complete elastic response (with no interaction between n and m increments) as the entire section is in initial elastic state. Two factors account for the degeneration of the subsequent elastic region: Firstly, according to 2SM-SS, the elastic range of steel fibers on the cross section shrinks with the increase of EPS (the effective plastic strain), thus the size of the elastic region for the entire section is limited by its most strained fibers. For beam-column cross section subjected to combined bending curvature and axial strain, the most strained fibers are the outmost fibers of the section. In the formulation, the straining of the outmost fibers is traced step by step and size of the elastic region is determined precisely. The other factor that contributes to the elastic-region degradation is change of loading path direction. For example, the cross section is strained from initial state to yield plateau state yield under a proportional loading path (the ratio of incrementnal axial strain to the incremental bending curvature $d\varepsilon / d\phi$ remains constant), and suppose both sides of the outmost fibers have yielded. Only a complete unloading path (along the original loading path with the same $d\varepsilon / d\phi$) is guaranteed to have complete elastic response, since all the fibers will be in elastic unloading, and change of loading path direction generally will trigger elasto-plastic response of cross section since some fibers will have elasto-plastic response. Considering these, the elastic region is finally degraded from the initial square area to a one-dimensional region (under a consistently proportional loading path) or even no elastic region at all (under irregular loading paths).

The second important feature of the cross-sectional behavior−a yield plateau stage is directly related to the existence of yield plateau with the structural steel material. Likewise, there will be a yield plateau stage under pure bending and under a general monotonic proportional loading path. During the yield plateau stage, further loading will cause little change in both stress resultants N and M since almost all the fibers are in yield plateau stage. This phenomenon receives a special treatment of trivial stiffness matrix in the current formulation.

Since the elastic region of the cross section degenerates with plastic deformation, there is no subsequent loading curve in this formulation. As a result, the plastic flow direction is not immediately obtainable through classic plasticity theory of associated flow type. Instead, approximation of realistic plastic flow direction is used in deriving the incremental stiffness matrix. The plastic flow direction reflects the relative magnitudes of the incremental plastic deformations ($d\varepsilon_p$ and $d\phi_p$) during the plastic loading. Realistic plastic flow direction means that one abstracted from fiber analysis at a specific loading point. Information from fiber analysis strongly suggests that force-space formulation follows quite different rules from the usual stress-space formulation in this respect, thus it seems inappropriate as well as impossible to use a similar approach. As mentioned already, the stress and strain at both sides of extreme fibers are monitored throughout, and the plastic flow direction is calculated step by step based on the state of the extreme fibers as well as the loading conditions of the entire cross section.

3. Preliminary Results

The results from the Force-space formulation under four selected monotonic proportional loading paths ($d\varepsilon / d\phi$ = 0.183H,0.365H,0.585H and 1.316H, H= cross section height) are shown in Fig.3a-c in comparison with the fiber analysis results using 2SM-SS, and the cross section concerned is the circular section of SS400 steel with D/t= 49.6 . With some necessary calibration, the force-space results can be in good agreement with the elaborate fiber analysis results.

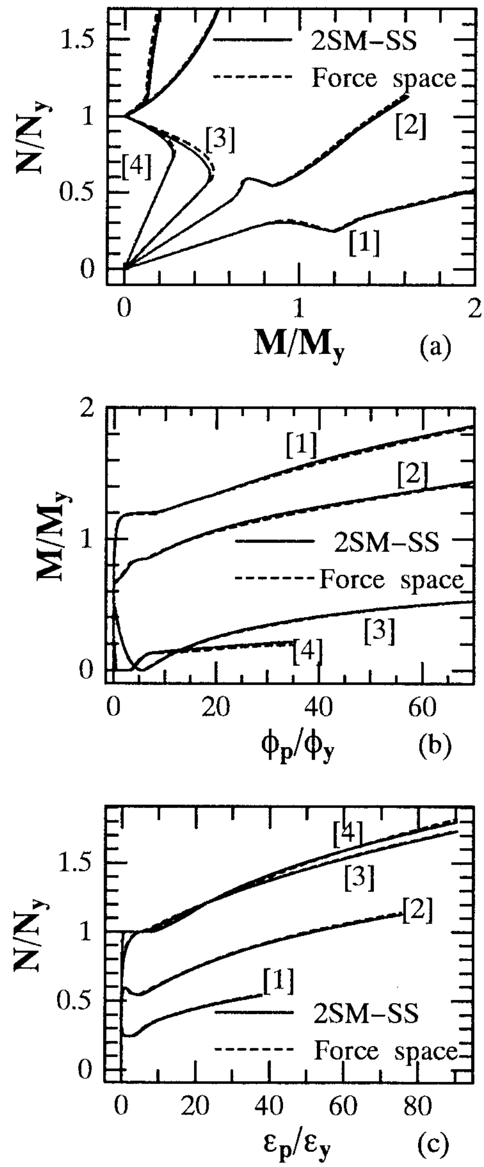

Fig.3 Comparison of Force-space results with fiber analysis results

4. Concluding Remarks

An attempt has been made to simulate the interaction of stress-resultants on the beam-column cross section by force-space plasticity. This force-space formulation is developed based on a precise stress-space plasticity model, and the results are compared with the fiber analysis results using the stress-space model. It has been found that realistic plastic flow direction and substantial calibration work are needed for the force-space formulation to produce accurate results.

References

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