# Application of Unified Concrete Plasticity Model in Reinforced Concrete Members

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Three Dimensional finite element analysis of Reinforced Concrete (RC) members is a very complicated matter. Recently a plasticity model, named Unified Concrete Plasticity Model which was first developed by Tanabe et. al.[1], was further developed by the present authors[2,3] which can be applied in multi-axial stress-strain situation. In this paper, applicability of this model is shown by applying it to a finite element analysis of a beam, under two point loading

## 1.INTRODUCTION

The initial yield surface of the unified concrete plasticity model is shown in Figure 1. In this paper, the first order yield function and the simple damage law $d\omega = \beta' d\varepsilon^p = \beta' d\lambda$ is being assumed. The Yield function is given as

$$g(\sigma,\omega) = \sqrt{J_2 + k_f(1 - AA^* / \eta_0)^2} - (k - \alpha_f I_1) = 0 \qquad (1)$$

where $I_1, J_2, J_3$ are stress invariant and

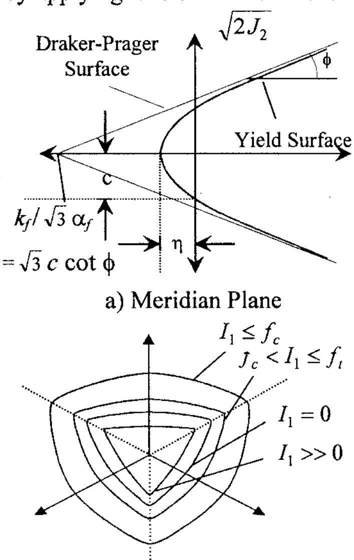
$$k_f = \frac{6C\cos\phi}{\sqrt{3}(3 + y\sin\phi_1)}, \alpha_f = \frac{2\sin\phi}{\sqrt{3}(3 + y\sin\phi_1)} \qquad (2)$$

where $\phi_1 = 14°$ is a material constant ,

$$y = \sqrt{a(\cos 3\theta + 1.00) + 0.01} - 1.10, \quad a = 0.5r^2 + 2.1r + 2.2, \quad r = \begin{cases} 3.14 & I \le f_c' \\ 2.93\cos\left(\dfrac{f_t - I_1}{f_t - f_c'}\pi\right) & f_c' < I \le f_t \\ 9.0 & I > f_t \end{cases} \qquad (3)$$

with $\cos 3\theta = (3\sqrt{3}J_3) / (2\, J_2^{1.5})$,

$\eta = AA^* \sqrt{3}C\cot\phi$, $AA^* = \sqrt{3}c_o\cot\phi_o / \eta_o$. Cohesion C and friction angle $\phi$ depends on the damage $\omega$ and stress parameter $X\left(= I_1 / \sqrt{3J_2}\right)$ to get independent behavior of C and $\phi$ for uniaxial tension and uniaxial compression and are defined by material constants $\phi_0$, $\phi_f$, $c_0$, $\eta_0$ and $\kappa$=$10^{-3}$




b) Deviatoric Plan

Fig.1 The Initial Yield Surface

$$C = \gamma C_0 \exp\left[(-m_1\omega)p_1(X) + \left(-m_2^2\omega^2\right)p_2(X)\right] + (1-\gamma)C_0, \quad \phi = \begin{cases} \phi_0 + (\phi_f - \phi_0)\sqrt{(\omega+k)(2-\omega-k)}\,p_2(X) & \omega \le 1 \\ \phi_0 + (\phi_f - \phi_0)p_2(X) & \omega > 1 \end{cases} \qquad (4)$$

Input parameters have to be carefully selected because we can get same set of uniaxial tension and compression. The parameters were selected to match the Kupfer's experimental results and other criteria.

## 2. SIGNIFICANCE OF STRESS-STRAIN OF CONCRETE IN SMEARED CRACK ANALYSIS

For simplicity let us take into consideration the following cases where the stress in concrete is apparently in uniaxial tension. In case of plain concrete under uniaxial tension, generally the specimen fails by a single localized crack. However, uniaxial tension experiment on thin reinforced concrete panel with both lateral and longitudinal reinforcement where the force was applied through the reinforcements generates multiple cracks. Concrete between the cracks take significant amount of stress causing *tension stiffening effect.* In this case, as is well known, we can adopt the same stress-strain diagram of the unreinforced concrete and use bond elements which makes that analysis complicated. Various researchers have suggested that one should incorporate tension stiffening effect by adopting an appropriate stress strain diagram for concrete and reinforcement in tension. In this paper, model proposed by Thomas T.C.Hsu[4] is applied to a beam under two point loading. Appropriate parameters are assumed to get concrete stress strain relation similar to $\sigma_r = f({}_t\varepsilon_{cr} / \varepsilon_r)^{0.4}$ within reasonable limits. For uniaxial tension stress-strain relation of reinforcement, the crack is assumed to be perpendicular to the reinforcement for simplicity and

$$f_s = E_s\varepsilon_s \quad \varepsilon_s < \varepsilon_n \quad , \quad \frac{f_s}{f_y} = 0.93 - 2B + 0.25B\frac{\varepsilon_s}{\varepsilon_y} \quad \varepsilon_s > \varepsilon_n \quad , \quad B = \frac{1}{\rho}\frac{f_{cr}}{f_y} \qquad (5)$$

### 3. ANALYSIS OF BEAM

Figure 2a shows a beam under two point loading with stirrups to cause bending failure. The cylinder test showed $f_c$=33.75 Mpa. Based on this, $E_c$ and $f_t'$ (mean value) are chosen according to the CEB-FIP Model Code -90. Material parameters are chosen to satisfy Kupfer's experimental results and stress strain curve of concrete under tension to simulate tension stiffening effect. $E_c$=34600 MPa, $\mu$=0.22, $f_t'$=3.15 MPa, $f_c$=33.75 MPa, $\phi_1=14.3°$, $\phi_o=5°$, $\phi_o=36°$, $m_1$=4.0, $m_2$=0.83, $\eta_o$ =7.0 MPa, $k$ = 1.0 x $10^{-3}$, $\omega_f$=1.0, $\beta'=35$ and $\gamma$=0.92. Only quarter section is taken for analysis. X or Y direction displacements are restrained for all points in the two symmetry section as necessary. In the analysis, all sections are assumed to have uniform lateral reinforcement as a first stage of analysis.

The tension stiffening effect is incorporated based on the proposal by Hsu et.al. His experimental results are based on experiments on thin panels where the full sections is almost under uniform uniaxial tension. However, in case of a beam, cracks gradually propagate. Moreover, only a part of the section is under tension with non-uniform tensile strain distribution of tension. Hence the applicability of the model, directly, is a big question.

| Case | ρ | B | $f_y^*$ (Mpa) | $E^*$ |
|---|---|---|---|---|
| A | 0 | - | 370 | 0.0001 |
| B | 2.5 | 0.0314 | 323.4 | 0.00785 |

Table 1. Parameters for Reinforcement

Two cases, as shown in table 1 above, are considered in the analysis. In compression, the no tension stiffening effect is introduced. We notice that the results are quite in acceptable range. Since the number of cracks increase gradually, as crack propagates, it looks that the implementation of tension stiffening effect should be studied in more details.

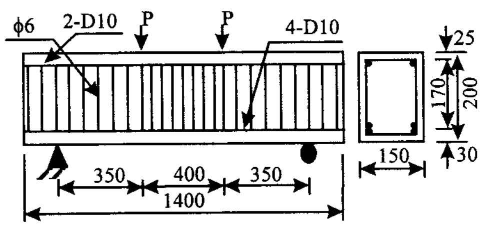


Fig. 2a. A Beam Under Two Point Loading

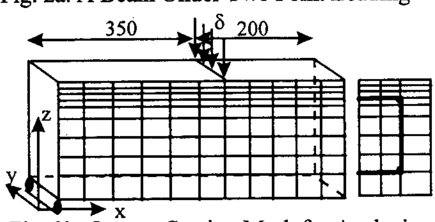


Fig. 2b. Quarter Section Mesh for Analysis

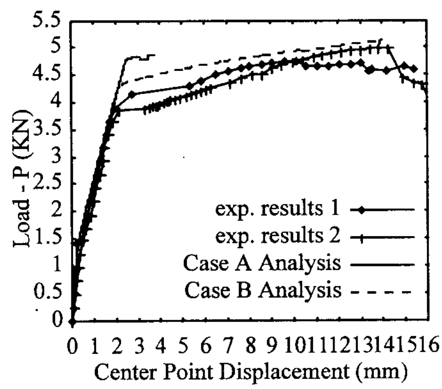


Fig. 2c. Load Deflection of Experiment and Analysis

### 4. CONCLUSION

The Modified Unified Concrete Plasticity Model is applied to the finite element analysis and found that this model can be reasonably used for finite element analysis. This is to note that, the original model which was presented was a second order yield surface encountered a lot of problems due to the deviation from the yield surface. However in the present first order yield surface, deviation for the yield surface in this analysis was under reasonable limit.


### REFERENCES

1. Tanabe, T., WU Z. and YU G., "A Unified Plastic Model for Concrete", JSCE, Vol. 24, No. 296, Aug., 1994, pp.21-29.
2. Gupta, S. and Tanabe, T., "Investigation and Modification of the Characteristics of the Unified Concrete Plasticity Model",JCI,1995,Vol.17.No.2,pp.1305-1310.
3. Gupta, S. and Tanabe, T., "Modified Unified Concrete Plasticity Model and its variations",JCI,Vol.18, No.2.,1996,pp.425-430.
4. Thomas T.C.Hsu, "Unified Theory of Reinforced Concrete",CRC Press.