リニアプログラミングモデルを用いた医療施設の配置計画問題
A Linear Programming Model for Allocating Health Care Facilities

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I. Introduction

In developing countries the efficient and loss costly provision of health care facilities is an important public policy. This paper aims at proposing an mathematical programming approach to the allocation of hierarchy systems of health care facilities. Basic idea is as follows: a successively inclusive facility hierarchy is a facility at level q provides all lower order services offered by a facility at level q-1 (i.e., level 1 through q-1) plust at least one additional service (Daskin,1995). For example, a typical health care system is summarized in table 1.

Table 1. Summary of Hypothetical Hierarchical Health Care System

	Service Provided		
Facility	Basic Care	Diagnostic Service	Out-Patient Surgery
Clinic	Yes	Yes	-
Hospital	Yes	Yes	Yes

Figure 1 illustrates the flow of patients in such a system.

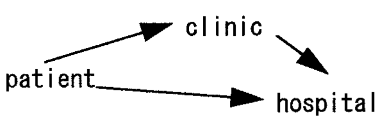

figure 1. Patient flows in hypothetical system.

Our objective is to minimize the total (over all village) demand weighted average distance between a village and the nearest health care facility to the village.

II. A Model

To illustrate how such this system can be modeled, we define the following notation:

Inputs

H=maximum number of hospital to be located, C= maximum number of clinics to be located, dij=distance between village i and candidate hospital j, dik=distance between village i and candidate clinic k, hi=population in village i, djk=distance between candidate hospital j and candidate clinic k, Dhc=critical coverage distance between hospitals and clinics.

$$a_{ij} = \begin{cases} 1: \text{if candidate clinic } k \text{ is within } D_{hc} \text{ distance} \\ \quad \text{units of candidate hospital } j \\ 0: \text{if not} \end{cases}$$

Decision Variables

$$X_j = \begin{cases} 1: \text{if candidate hospital} \\ \quad \text{site } j \text{ is selected} \\ 0: \text{if not} \end{cases}, \qquad Y_k = \begin{cases} 1: \text{if candidate clinic} \\ \quad \text{site } k \text{ is selected} \\ 0: \text{if not} \end{cases}$$

$$W_{ij} = \begin{cases} 1: \text{if patients at village} \\ \quad \text{site } i \text{ go to a hospital at} \\ \quad \text{candidate site } j \\ 0: \text{if not} \end{cases} \qquad V_{ik} = \begin{cases} 1: \text{if patients at village} \\ \quad \text{site } i \text{ go to a clinic} \\ \quad \text{at candidate site } k \\ 0: \text{if not} \end{cases}$$

With this notation, we can formulate the following demand-weighted average distance between a village and the nearest health care facility to the village:

$$\text{Minimize} \quad \sum_i \sum_j h_i d_{ij} W_{ij} X_j + \sum_i \sum_k h_i d_{ik} V_{ik} Y_k \qquad (1.a)$$

$$\text{Subject to} \quad \sum_j X_j \le H \qquad (1.b)$$

$$\sum_k Y_k \le C \qquad (1.c)$$

$$\sum_j W_{ij} + \sum_k V_{ik} = 1 \qquad \forall i \qquad (1.d)$$

$$Y_k \le \sum_j a_{jk} X_j \qquad \forall k \qquad (1.e)$$

$$W_{ij} - X_j \le 0 \qquad \forall i,j \qquad (1.f)$$

$$V_{ik} - Y_k \le 0 \qquad \forall i,k \qquad (1.g)$$

$$X_j, Y_k = 0,1 \qquad \forall j,k \qquad (1.h)$$

$$W_{ij}, V_{ik} = 0,1 \qquad \forall i,j,k \qquad (1.i)$$

The objective function (1.a) minimizes the total demand-weighted distance between a village and the nearest health care facility. Constraints (1.b) stipulate that the hospital must be located no more than H hospitals. Constraints (1.c) state that no more than C facilities of clinics are to be located. Constraints (1.d) requires each node i to be assigned to exactly one facility. Constraints (1.e) stipulate that each clinic must be within Dhc distance units of the nearest hospital. Constraints (1.f) state demand at village i can only be assigned to a hospital at candidate site j if we locate a

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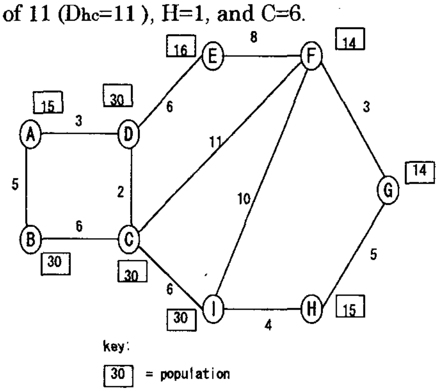
hospital at candidate site j. Constraints (1.g) similar to constraints (1.f) state that demands at node i can only be assigned to a facility at candidate clinic k (V_{ik}=1). Constraint (1.f) and (1.g) link the location variables (X_j,Y_k) and allocation variables (W_{ij},V_{ik}). Finally, constraints (1.h) and (1.i) are the integrality constraints, respectively.

III. Column-Row Reduction Rule for Solving the Problem

We try to formulate the problem for the network shown in figure 2. For a critical coverage distance between hospital and clinics of 11 (D_{hc}=11), H=1, and C=6.



distance value approximately to 8 km

link distance not drawn to scale

Figure 2. network illustration.

By the objective function and constraints (eqs.1), we will try to calculate this problem as follows,

MINIMIZE $\{h_A d_{AA} W_{AA} X_A + \ldots + h_I d_{II} W_{II} X_I\} +$

S.t. $\{h_A d_{AA} V_{AA} Y_A + \ldots + h_I d_{II} V_{II} Y_I\}$ (2.a)

$X_A + X_B + X_C + X_D + X_E + X_I \geq Y_A$ (2.b)

$X_A + X_B + X_C + X_D + \geq Y_B$ (2.c)

$X_A + X_B + X_C + X_D + X_E + X_F + X_H + X_I \geq Y_C$ (2.d)

$X_A + X_B + X_C + X_D + X_E + X_I \geq Y_D$ (2.e)

$X_A + X_C + X_D + X_E + X_F + X_G \geq Y_E$ (2.f)

$X_C + X_E + X_F + X_G + X_H + X_I \geq Y_F$ (2.g)

$X_E + X_F + X_G + X_H + X_I \geq Y_G$ (2.h)

$X_C + X_F + X_G + X_H + X_I \geq Y_H$ (2.i)

$X_A + X_C + X_D + X_F + X_G + X_H + X_I \geq Y_I$ (2.j)

$X_A + X_B + X_C + X_D + X_E + X_F + X_G + X_H + X_I \leq 1$ (2.k)

$Y_A + Y_B + Y_C + Y_D + Y_E + Y_F + Y_G + Y_H + Y_I \leq 6$ (2.l)

$X_A, X_B, X_C, X_D, X_E, X_F, X_G, X_H, X_I = 0, 1$ (2.m)

$Y_A, Y_B, Y_C, Y_D, Y_E, Y_F, Y_G, Y_H, Y_I = 0, 1$ (2.n)

We can reduce the size of the problem using a variety of reduction rules (Daskin, 1995 p. 95-96). We begin with a column reduction rule. Consider two columns p and q. If $a_{ip} \leqq a_{iq}$ for all demands nodes i and $a_{ip} \leqq a_{iq}$ for at least one demand node i, the location q covers all demand covered by location p. We say that location q *dominates* location p. In addition, we can then set X_p=0.

Next, we consider second row reduction rule (Daskin, 1995 p.98) which allow us to eliminate rows from the problem. This rule allow us to eliminate the rows corresponding to nodes A, B, C, D, F, H, and I. After the row reductions, the constraints of the problem becomes

(Node E Coverage) $X_C + X_E + X_F \geq Y_E$ (3.a)

(Node G Coverage) $X_E + X_F + X_I \geq Y_G$ (3.b)

(No.to Locate) $X_C + X_E + X_F + X_I \leq 1$ (3.c)

The resulting solution is,

$X_C = 1, X_A = X_B = X_D = X_E = X_F = X_G = X_H = X_I = 0$

$Y_B = Y_D = Y_E = Y_F = Y_H = Y_I = 1$, $Y_A = Y_G = 0$ or

$Y_A = Y_B = Y_E = Y_F = Y_H = Y_I = 1$, $Y_D = Y_G = 0$.

This means that clinics will be located at site B, D, E, F,H, I or A, B, E, F, H, I. Finaly, the total objective function is 42+1123=1165.

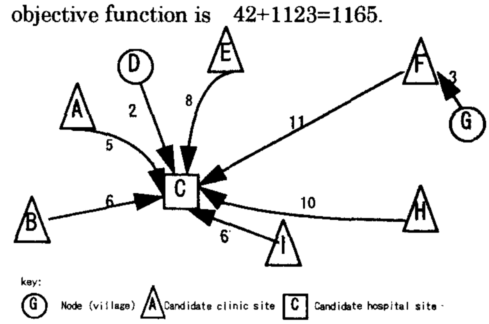


Figure 3. One alternative of patient flows.

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