MODELLING AND ANALYSIS OF MASONRY STRUCTURES

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Introduction: The present paper is concerned with the analysis of masonry structures consisting of distinct blocks. A technique based on the finite element method together with the contact element to model structures consisting of blocks of arbitrary shapes is developed and employed in the analysis. The developed technique is called the Discrete Finite Element Method (DFEM) and considers deformable blocks as sub-domains and represent them by solid elements. The block interaction, such as sliding or separation, is modeled with the contact elements which are far-superior to the joint or interface elements. The developed DFEM is used to simulate behavior of masonry engineering structures, such as, a single block or a piles of blocks on an incline, arch structures and masonry dams. Some of the results have been compared with those obtained from other techniques. It is shown that the proposed approach is very promising to analyze masonry structures.

Finite element modeling of intact blocks: Small displacement theory is applied to the intact blocks while blocks can take finite displacement. Blocks are polygons with an arbitrary sides which are in contact with the neighboring blocks and are idealized as a single or multiple finite elements. The generalized finite element form of the equation of motion using the displacement approximation of $u = NU(t)$ and constitutive law can be obtained as (Mamaghani, 1993):

$$M\ddot{U} + C\dot{U} + KU = F \tag{1}$$

in which,
$M = \int_{\Omega_e} \rho\, N^T N d\Omega$; $C = \int_{\Omega_e} B^T D_v B d\Omega$; $K = \int_{\Omega_e} B^T D_e B d\Omega$; $F = \int_{\Omega_e} N^T b d\Omega + \int_{\Gamma_{te}} \bar{\mathbf{N}}^T \mathbf{t} d\Gamma$. Here, $U, N, B, b, \rho, \ddot{U}$ denote nodal displacement vector, shape function, strain-displacement matrix, body force, density and acceleration respectively. D_e and D_v are elasticity and viscosity tensors (Aydan et al. 1995).

Finite element modeling of block contacts: Let us now consider a two-nodded element (l, m) in a two-dimensional space and take two coordinate systems (oxy) and $(o'x'y')$ as shown in Fig. 1. Assuming that, the strain component $\varepsilon_{y'y'}$ is negligible, the remaining strain components take the following form:

$$\varepsilon_{x'x'} = \frac{\partial u'}{\partial x'}, \quad \gamma_{x'y'} = \frac{\partial v'}{\partial x'} \tag{2}$$

Let us assume that the shape functions are linear such that:

$$N_l = \frac{1}{2}(1-\xi), \quad N_m = \frac{1}{2}(1+\xi), \quad \xi = (2x' + x'_l + x'_m)/L \tag{3}$$

where, $L = (x'_l - x'_m)$. Then, the relation between the strains and nodal displacements becomes

$$\left\{ \begin{array}{c} \varepsilon_{x'x'} \\ \gamma_{x'y'} \end{array} \right\} = \frac{1}{L} \begin{bmatrix} -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \left\{ \begin{array}{c} U'_l \\ V'_l \\ U'_m \\ V'_m \end{array} \right\} \tag{4}$$

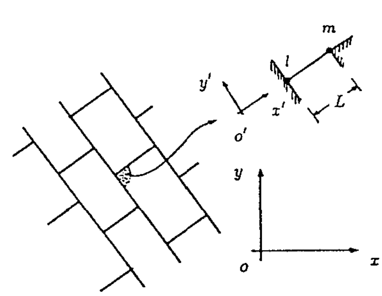

Fig. 1 Finite element representation of a contact element

Thus, the stiffness matrix in the local coordinate system is obtained in an explicit form as

$$K' = \begin{bmatrix} k'_n & 0 & -k'_n & 0 \\ 0 & k'_s & 0 & -k'_s \\ -k'_n & 0 & k'_n & 0 \\ 0 & -k'_s & 0 & k'_s \end{bmatrix}, \quad k'_n = E_t \cdot \frac{A_c}{x'_m - x'_l}, \quad k'_s = G \cdot \frac{A_c}{x'_m - x'_l} \tag{5}$$

in which A_c is the contact area. Note that k'_n and k'_s involve the material properties and geometry of contact zone. Direct shear tests will yield the normal stiffness and shear stiffness, directly without requiring information about contact conditions. The details of mechanical and finite element formulation are given in a work by Mamaghani 1993.

Numerical results and discussions: The numerical results reported are two dimensional and elliptic type with $M = 0$, $C = 0$ and $KU = F$. The material properties of intact blocks (Young Modulus $E = 50GPa$, Poison's ratio $\nu = 0.2$, and $\rho = 25kN/m^3$) and the properties of contacts (normal stiffness $E_n = 50GPa$ and shear stiffness $G_s = 0.5GPa$) used in all numerical analysis reported here were same.

One block on an incline: A very simple, yet meaningful problem that can be analyzed by the proposed DFEM is the stability of a rectangular block on an inclined plane. The theoretical kinematics conditions for sliding and toppling of a single block on an incline, under gravity, with the friction angle between the block and the incline $\phi = 20°$ is shown in Fig. 2. Four modes of behavior, namely, (a) stability, (b) sliding without toppling, (c) sliding and toppling, and (d) toppling without sliding are delineated by four boundaries (I, II, III & IV). For a methodical comparison, the slope angle α and the aspect angle γ were varied systematically, while ϕ was fixed at 20°. The results by the DFEM, plotted using different symbols representing different modes of behavior, are in complete agreement with the theoretical results, as can be seen in Fig. 2.

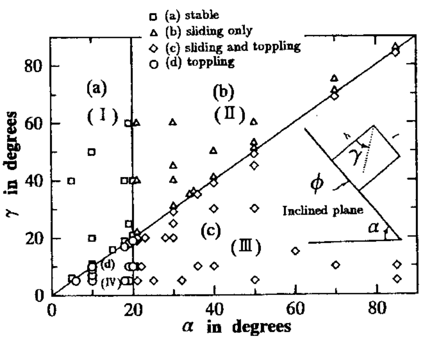

Fig. 2 Comparison of numerical results for a block on an incline with a Hoek-Bray chart

Fig. 3 depicts the numerical results by the DFEM for a system of three blocks sliding down on an incline, under its own weight, with $\gamma = 42° > \alpha = 40° > \phi = 35°$. The three blocks are sliding over each other from top to bottom.

Arch structure: Fig. 4 shows an arch structure analyzed using the proposed DFEM. The friction coefficient between blocks is $\mu_s = 0.7$. When the dead weight of the blocks are applied, the arch is stable. It is still stable when the distributed traction is less than $150kgf/m^2$. However, if the traction is reaches that level, then the arch starts to be unstable. Fig. 4 shows the configuration of the arch at different iterations.

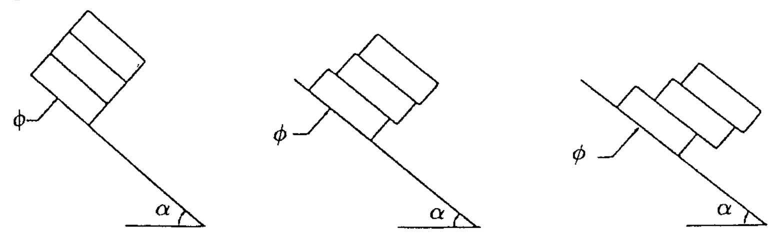

Fig. 3 Failure mechanism of a three blocks system on an incline

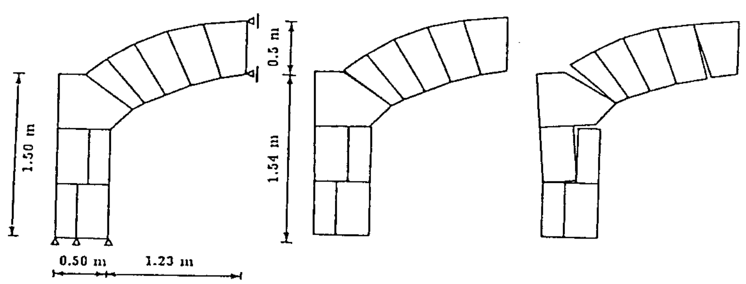

Fig. 4 Failure mechanism of an arch

Conclusions: A technique based on the finite element method together with contact element modeling block interaction to analyze masonry structures has been developed. The proposed scheme is used to follow the process of failure mechanism of blocky structures. However, it is in its formative phase and further research work is necessary to model complex failure behavior of rock block systems.

References: (1) Mamaghani I.H.P.: "Numerical analysis for stability of a system of rock blocks." *Master Thesis, Dept. of Civil Engineering, Nagoya University*, 1993. (2) Aydan et al.: "Prediction of deformation behavior of a tunnel in squeezing rock with time dependent characteristics." *5th Int. Symp. on Numerical models in Geomechanics, NUMOGV, 463-469*, 1995.