A Circular Rigid Punch with Two Ends in Smooth Contact with an Edge Cracked Semi-infinite Plane Acted by Concentrated Forces

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1. Introduction

A circular rigid punch with sharp corners in complete contact with a cracked half plane has been studied before[1], and the fundamental solution of the problem in [1] has also been derived recently[2]. The present paper deals with the problem of a circular rigid punch in smooth contact with an edge cracked semi-infinite plane acted by a pair of concentrated forces at an arbitrary position. The punch is acted by a vertical load with a distance from the origin of the coordinates to keep the punch not to incline, and there exists frictional force on the contact region. The problem is solved by mapping the semi-infinite plane with the edge crack into the unit circle, and establishing the corresponding Riemann-Hilbert equations. The influences of the load on the punch, the radius of curvature of the punch and the concentrated forces in the semi-infinite plane are reflected by separated terms in the general solution of the problem. The smooth contact conditions at the two smooth ends are formed and used to determine the positions of the two contact ends, then the contact length can be decided. The stress distribution on the contact region is shown with different frictional coefficients.

2. The presentation of the problem

As shown in Fig.1, the punch is supposed to be vertical on a half plane with an edge crack. To keep the punch not to incline, the position of the load on the punch is usually eccentric from the origin of the x-y coordinates to equilibrium the moment produced by the stresses on the contact region about the origin. Besides the load P on the punch, there also exists a pair of concentrated forces at point z_0 in the half plane. To solve the problem in an analytical way, the cracked half plane is mapped into a unit circle by the following rational mapping function[1]:

$$z = \omega(\zeta) = \frac{E_0}{1-\zeta} + \sum_{k=1}^{N} \frac{E_k}{\zeta_k - \zeta} + E_c \tag{1}$$

where E_0, E_k, ζ_k are known coefficients, and E_c is decided by the distance from the edge crack to the punch.

The loading and displacement conditions can be described as

$$p_x = p_y = 0 \quad \text{on} \quad L = L_1 + L_2 \tag{2a}$$

$$p_x = \mu p_y, \quad \int p_y ds = P \quad \text{on} \quad M \tag{2b}$$

$$V = x^2 / 2R \quad \text{on} \quad M \tag{2c}$$

$$Q(x,y) = (q_x + iq_y)\delta(z, z_0)$$

where $L_1 = ABCD'D, L_2 = EA, M = DE$ in Fig.1, μ is the Coulomb's frictional coefficient on M; p_x and p_y represent the components of traction in x and y directions on the surface of the half plane; $Q(x,y)$ represents the concentrated forces on the boundary or in the body of the half plane; $\delta(z,z_0)=1$ when $z=z_0$ and 0 when $z \neq z_0$. V is the displacement of the punch, and R is the radius of curvature.

3. The method of analysis

The complex stress functions of the problem are divided into two parts:

$$\phi(\zeta) = \phi_1(\zeta) + \phi_2(\zeta) \tag{3a}$$

$$\psi(\zeta) = \psi_1(\zeta) + \psi_2(\zeta) \tag{3b}$$

where $\phi_1(\zeta)$ and $\psi_1(\zeta)$ are the complex stress functions of the half plane with the edge crack acted by the concentrated forces[2]. $\phi_2(\zeta)$ and $\psi_2(\zeta)$ are the holomorphic parts of $\phi(\zeta)$ and $\psi(\zeta)$.

The general solution of $\phi_2(\zeta)$ can be expressed as [1,2]

$$\phi_2(\zeta) = H_1(\zeta) + H_2(\zeta) + H_3(\zeta) + \frac{1+i\mu}{2} J(\zeta) + Q(\zeta)\chi(\zeta) \tag{4}$$

where $H_1(\zeta), H_2(\zeta), Q(\zeta)\chi(\zeta)$ and $J(\zeta)$ are the same expressions as those in the previous paper[1], and $H_3(\zeta)$ is related to the concentrated forces in the half plane, which is expressed as

$$H_3(\zeta) = \frac{1-i\mu}{4\pi}\left[(\bar{q} - \kappa q)F_1 + (\kappa\bar{q} - q)F_2 + qG_1 + \bar{q}G_2 + 2\pi G_3\right] \tag{5}$$

where F_1, F_2, G_1, G_2 and G_3 can be found in [2]. $q = -(q_x + iq_y)/(1+\kappa)$, $\kappa = 3-4\nu$ for plane strain and $(3-\nu)/(1+\nu)$ for plane stress state, and ν is the Poisson's ratio of the half plane.

4. The conditions of smooth contact

The stress components on the boundary of the half plane can be expressed as

$$\sigma_r + i\tau_{r\theta} = \frac{1}{\omega'(\sigma)}\left\{\frac{i(1-i\mu)P}{2\pi}\frac{(1-\alpha)(1-\beta)}{\chi(1)(1-\sigma)(\sigma-\alpha)(\sigma-\beta)} + \frac{e(\sigma)}{(\sigma-\alpha)(\sigma-\beta)} + \frac{mf(\sigma)}{\sigma-\alpha} + \frac{(1-m)f(\sigma)}{\sigma-\beta} + g(\sigma)\right\} \times \left[\chi^+(\sigma) - \chi^-(\sigma)\right] \tag{6}$$

where $e(\sigma), f(\sigma)$ and $g(\sigma)$ are related to the concentrated forces in the half plane, which are known functions.

The following conditions can then be formed to satisfy the finite stress condition at the smooth ends:

$$\frac{i(1-i\mu)(1-\beta)}{2\pi\chi(1)(\alpha-\beta)}P + \frac{e(\alpha)}{\alpha-\beta} + mf(\alpha) = 0 \tag{7a}$$

$$\frac{i(1-i\mu)(1-\alpha)}{2\pi\chi(1)(\beta-\alpha)}P + \frac{e(\beta)}{\beta-\alpha} + (1-m)f(\beta) = 0 \tag{7b}$$

α and β can be obtained from (7), and then the length of the contact region can be decided. The stress distribution on the contact region can then be obtained. Fig.2 shows the stress distributions on the contact region with different frictional coefficients. The concentrated forces are typically taken as $q_y / P = 1$ and $q_x = 0$ acted at $(0, -b)$ and the special case of $q_x = q_y = 0$. $\kappa = 2$, $d/b = 2$ and $Gb^2/PR = 1$ were selected. The stress intensity factors of the edge crack and the resultant moment on the contact region can also be calculated, which are omitted here owing to the limited length of the paper.

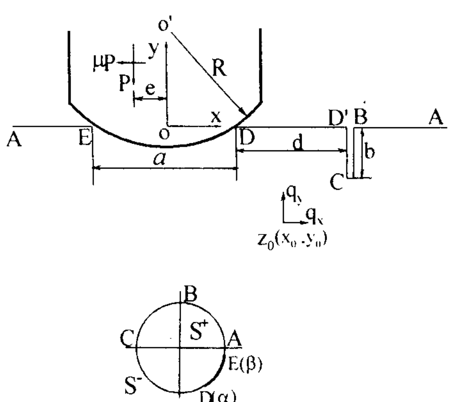

Fig.1 The punch and the unit circle

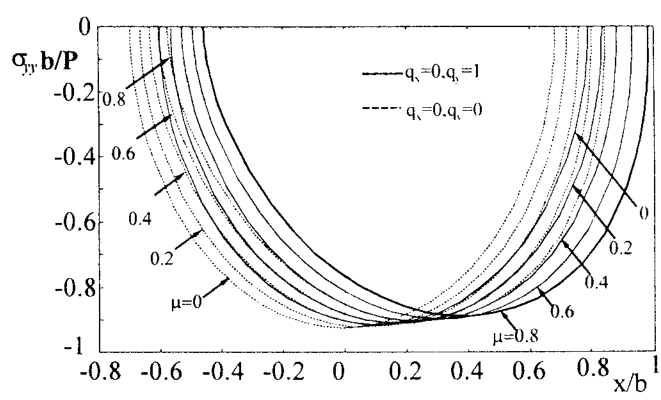

Fig.2 The stress distribution on the contact region

References

[1] Hasebe N. and Qian J., Contact Mechanics II, Southampton (1995)

[2] Qian J. and Hasebe N., 11th International Conference of BEM, Hawaii(1996)