Local Iteration Method And Its Application in Consolidation Problem

Nagoya University Student member Jian-Guo WANG
Nagoya University Member Takafumi SEIKI
Nagoya University Member Yasuaki ICHIKAWA

1. The problem

Nonlinear or elastoplastic FEM is always carried out by an incremental step-by-step method. The frequently used method is the Newton-Raphson method and its improvements. This method has three important properties.

1, $[D]_{ep}$ is an approximation when it is linearized.
2, The iteration is carried out for all variables.
3, The variable renewal is iteration path-dependence.

In order to improve the convergence and reduce the computation time a new method called a Local Iteration Method (Wang 1996) was put forth. Fig.1 is the comparison between usual elastoplastic FEM procedure and the Local Iteration Method.

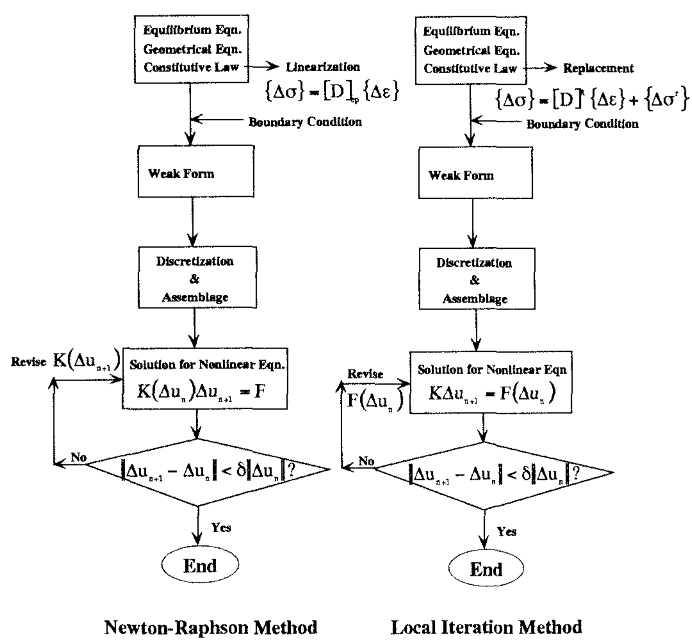

Fig.1 Comparison between two methods

In fact, two notes should be made on the mathematical properties of an initial value problem. a) Governing equations can be classified into two classes: one is a global equation which involves equilibrium equation and geometrical equation. The other is a local equation which is constitutive law. b) The global equation is linear in infinitesimal deformation, the local equation may be nonlinear. By using this mathematical properties a Local Iteration Method (LIM) is put forth. This method can include Initial stress/strain method. Following is the outline and its application in a nonlinear consolidation problem.

LIM is based on following two concepts:

Reference Constitutive Law

$$\{\Delta\sigma^*\} = [D]^R\{\Delta\varepsilon\} \quad \text{to replace the true constitutive law} \quad \{\Delta\sigma\} = [D]_{ep}\{\Delta\varepsilon\} \tag{1}$$

The error stress $\{\Delta\sigma^r\}$ induced by this replacement is

$$\{\Delta\sigma^r\} = \{\Delta\sigma\} - \{\Delta\sigma^*\} \tag{2}$$

Or the true constitutive is reformed as

$$\{\Delta\sigma\} = [D]^R\{\Delta\varepsilon\} + \{\Delta\sigma^r\} \tag{3}$$

Local Iteration in each element

Nonlinear iteration is limited to each element. The procedure is drawn out to find out the error stress $\{\Delta\sigma^r\}$

$$\{\Delta\varepsilon\} \quad \Longrightarrow \quad \{\Delta\sigma\} \quad : \quad \{\Delta\sigma^r\} = \{\Delta\sigma\} - \{\Delta\sigma^*\} \tag{4}$$

2. LIM Mode For Consolidation Problem

The fundamental equation of Biot's theory is composed of six concepts (Ichikawa 1990; Wang 1996). Its weak form is easily obtained from the virtual work principle

$$\begin{aligned}
&\int_V \{\delta(\Delta\varepsilon)\}^T[D]^R\{\Delta\varepsilon\}dv - \int_V \left\{\delta\left(\frac{\partial\Delta\bar{u}_i}{\partial x_i}\right)\right\}^T \{u\}^{t+\Delta t}dv - \int_V \{\delta(\Delta\bar{u})\}^T\{\Delta b\}dv \\
&+ \int_{S_\sigma} \{\delta(\Delta\bar{u})\}^T\{n\}u^{t+\Delta t}ds - \int_{S_\sigma} \{\delta(\Delta\bar{u})\}^T\{\Delta\bar{T}\}ds \qquad (5) \\
&= -\int_V \{\delta(\Delta\varepsilon)\}^T\{\sigma^t\}dv + \int_{S_\sigma} \{\delta(\Delta\bar{u})\}^T\{\bar{T}^t\}ds - \int_V \{\delta(\Delta\bar{u}\}^T\{b^t\}dv - \int_V \{\delta(\Delta\varepsilon)\}^T\{\Delta\sigma^r\}dv
\end{aligned}$$

The weak form of continuity equation is

$$-\int_V \{\delta u\}^T\left\{\frac{\partial\Delta\bar{u}_i}{\partial x_i}\right\}dv = \frac{1}{r_w}\int_t^{t+\Delta t}\left[\int_{S_q}\{\delta u\}^T\{Ku\}ds\right]dt - \frac{1}{r_w}\int_t^{t+\Delta t}\left[\int_V\{\frac{\partial\delta u}{\partial x_i}\}^T\{K_i\frac{\partial u}{\partial x_i}\}dv\right]dt \tag{6}$$

Eq.(5, 6) are the self-corrector mode of LIM method for consolidation problem. The error stress $\{\Delta\sigma^r\}$ is converted into a body force as $-\int_V\{\delta(\Delta\varepsilon)\}^T\{\Delta\sigma^r\}dv$. The key point is how to take $[D]^R$. For elastoplastic constitutive model

$$\{\Delta\sigma\} = [D]\{\Delta\varepsilon\} - d\lambda\{\sigma^g\} \tag{7}$$

$\boldsymbol{\sigma}^g$ is approximately known at the beginning of each step. $[D]$ is elastic matrix. The only unknown $d\lambda$ is pre-determined by Local Iteration in each element. Therefore, the global iteration is always linear and global stiffness may be constant in iteration procedure. Furthermore, a iteration path-independence variable renewal is adopted such as

$$\boldsymbol{u}_{n+1}^i = \boldsymbol{u}_n + \boldsymbol{\Delta u}^i \tag{8}$$

$\boldsymbol{u}_n$ is the precious value at n^{th} step. The iteration is numbered by i.

3. Examples (Omitted)

4. Conclusion

A completely different idea on nonlinear or elastoplastic FEM computation, Local Iteration Method is extended to consolidation problems. This method uses a approximate constitutive law, reference constitutive law, instead of the linearized one of true constitutive law. The weak form of a consolidation problem is developed to be suitable for Local Iteration Method. The computation of examples has shown that this extension is feasible and its computation can be reduced increasingly.

References

市川康明(1990), 地盤力学における有限要素法入門, 日科技連出版社

Wang, J.-G.(1996), *A Homogenization Theory for Geomaterials: Nonlinear Effect and Water Flow*, Dr.Eng Thesis, Dept. of Civil Engineering, Nagoya University