Thermal stresses around the thin circular disc-like inclusion in an infinite medium under general temperature conditions

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1. Introduction

This paper deals with the distribution of thermal stress around a penny-shaped crack of prescribed deformed shape under general temperature conditions. The displacement prescribed on the crack faces is unsymmetric about the plane z=0 and temperature on crack faces is distinct. Physically this is equivalent to a thin unsymmetrical disc-like inclusion situated in the plane z=0 and it is subjected to non-uniform temperature conditions. That is, the disc inclusion under non-uniform temperature adheres to the matrix on the upper surface $(z=0+, 0<r<a)$ differently from the lower surface $(z=0-, 0<r<a)$. The problem is solved using the method developed in Refs.[1,2]. Stress and displacement components and temperature field $\sigma_{zz}(r,z)$, $\sigma_{rz}(r,z)$, $u_r(r,z)$, $u_z(r,z)$, $\theta(r,z)$ in terms of stress, displacement, temperature and heat flux discontinuities at the crack plane are given in [1,2] (See equations (2.19)-(2.24) of Ref.[2] and (45)-(51) of Ref.[1]). The limiting values of these stress, displacement and temperature fields as $z \to 0+$ and as $z \to 0-$ can be used to solve the crack problem of prescribed face displacements defined in the next section.

2. Solution of the Penny-shaped crack problem

Let the penny-shaped crack be located in the plane z=0 of homogeneous isotropic infinite elastic body. Here we consider the problem of finding out the distribution of thermal stress acting on the faces of a Penny-shaped crack of prescribed shape subjected to non-uniform temperature conditions. In terms of the cylindrical coordinates (r, ϕ, z) the crack faces are defined as $(0<r<a, z \to 0+)$ and $(0<r<a, z \to 0-)$. The crack is assumed to be in a prescribed deformed shape which need not be symmetric about the plane $z=0$. The crack faces are subjected to general temperature, that is, temperature on two crack faces is distinct. The continuity and the boundary conditions may be written

$$\theta^{(1)}(r,0) = \theta^{(2)}(r,0), \qquad r > a \tag{1}$$

$$\frac{\partial}{\partial z}\theta^{(1)}(r,0) = \frac{\partial}{\partial z}\theta^{(2)}(r,0), \quad r > a \tag{2}$$

$$\theta^{(1)}(r,0) - \theta^{(2)}(r,0) = T_1^*(r), \qquad 0 \le r < a \tag{3}$$

$$\theta^{(1)}(r,0) + \theta^{(2)}(r,0) = T_2^*(r), \qquad 0 \le r < a \tag{4}$$

together with (5)-(12)of Ref.[3]. Using the Boundary conditions of the problem and eqn. (22)-(27), (75), (90)-(93) of Ref.[1] we get the expressions for the functions A, B, C, D, E, F in terms of the prescribed loading functions E^*, F^*, G^*, H^*, T_1^*, T_2^* and are given by

$$A(\rho) = 0,\ B(\rho) = 0,\ C(\rho) = 0, \qquad \rho > a \tag{5}$$

$$D(\rho) = 0,\ E(\rho) = 0,\ F(\rho) = 0, \qquad \rho > a \tag{6}$$

$$A(t) = \int_t^a \frac{[G^*(r) + rG^{*'}(r)]}{\sqrt{r^2 - t^2}} dr, \qquad 0 < t < a \tag{7}$$

$$B(t) = \int_t^a \frac{tE^{*'}(r)}{\sqrt{r^2 - t^2}} dr, \qquad 0 < t < a \tag{8}$$

$$F(t) = -[T_2^*(0) + \int_0^t \frac{tT_2^{*'}(r)}{\sqrt{t^2 - r^2}} dr], \qquad 0 < t < a \tag{9}$$

$$C(t) = -\frac{2\mu(\lambda+2\mu)}{(\lambda+3\mu)} \int_0^t \frac{[H^*(r) + rH^{*'}(r)]}{\sqrt{t^2 - r^2}} dr + \frac{2\mu^2}{(\lambda+3\mu)} B(t) + \frac{\mu(\lambda+2\mu)(1+\nu)\alpha}{(\lambda+3\mu)(1-\nu)} \int_0^t \frac{rT_2^*(r)}{\sqrt{t^2 - r^2}} dr, \quad 0 < t < a \tag{10}$$

$$D(t) = -\frac{2\mu(\lambda+2\mu)}{(\lambda+3\mu)} \left[F^*(0) + \int_0^t \frac{tF^{*'}(r)}{\sqrt{t^2 - r^2}} dr \right] - \frac{2\mu^2}{(\lambda+3\mu)} A(t) - \frac{2\mu(\lambda+2\mu)(1+\nu)\alpha}{(\lambda+3\mu)(1+\nu)} E(t), \quad 0 < t < a \tag{11}$$

where λ, μ are Lame's constants, ν, α are Possion's ratio and coefficient of linear expansion of the solid, respectively. Since the stress and displacement components are in terms of A, B, C, D, E, F, we can simplify them easily. In principle this completes the solution of the problem. Since the problem of determining the axially symmetric stress required to maintain penny-shaped crack of defined deformed shape has been solved [3], in the next section we shall consider the determination of stress distribution at the general point of the medium where the crack is subjected to only thermal conditions. In order to get the combined effect we can superpose the solutions.

3. Special case of temperature field on crack faces

Let the deformed crack faces are subjected to a constant temperature. The constant temperature applied on the upper surface (0<r<a, z=0+) of the crack be $(\tau_1 + \tau_2)/2$, while the temperature applied

on the lower surface (0<r<a, z=0-) of the crack be $(\tau_2-\tau_1)/2$. If we assume displacement conditions are zero, we have

$$E^*(r)=0;\ F^*(r)=0,\qquad 0<r<a \tag{12}$$

$$G^*(r)=0;\ H^*(r)=0,\qquad 0<r<a \tag{13}$$

$$T_1^*(r)=\tau_1;\ \ T_2^*(r)=\tau_2,\ \ 0<r<a \tag{14}$$

For this special case of temperature functions, expressions for stress at a general point of the medium are derived. Expressions for stress components contains functions in complex variable and integrals. When $\tau_1=\tau_0,\tau_2=\tau_0$ and $\lambda=\mu$, that is, $\nu=0.25$, complex functions in the stress components are computed using the results of Ref.[4] and integral are evaluated using Gauss-Chebyshev numerical integration rule. The variation of stress with spatial coordinates (r , z) presented in Fig.1-4. The non-dimensionalised parameter C_0 in Fig.1-4 is given by $C_0=-\mu(1+\nu)\alpha\tau_0/(1-\nu)$.

4. Concluding Remarks

1. Unlike the crack problem, in the case of the displacement boundary value problem, stresses at a general point of the medium are more complicated.
2. In similar lines we can also solve the crack problem of prescribed face displacement under general heat flux conditions.

5. References

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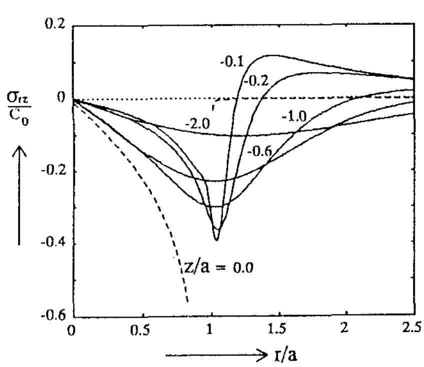

Fig.4 Variation of the stress component σ_{rz}/C_0 with r/a for z<0

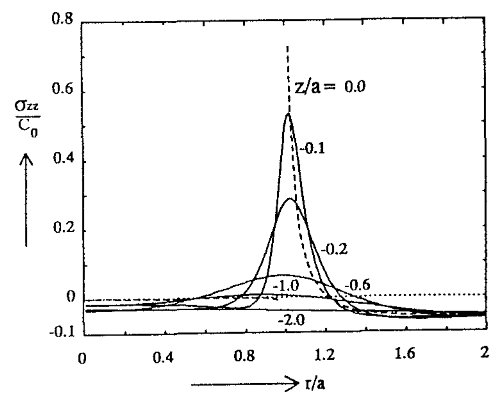

Fig.2 Variation of the stress component σ_{zz}/C_0 with r/a for z<0

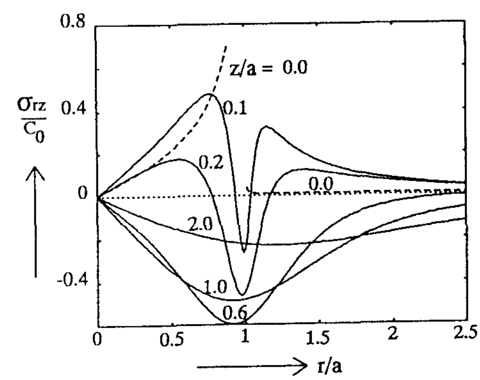

Fig.3 Variation of the stress component σ_{rz}/C_0 with r/a for z>0

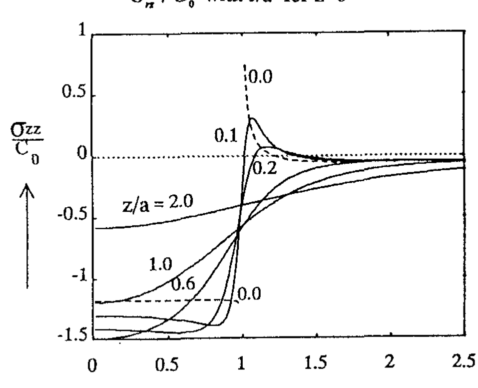

Fig.1 Variation of the stress component σ_{zz}/C_0 with r/a for z>0