THE STUDY OF SPACE STRUCTURES OPTIMIZATION BY MINIMIZING THE SQUARE SUM OF NATURAL PERIODS OF VIBRATION

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1. Introduction

This paper presents an optimization method in structural vibration by minimizing the square sum of natural periods of vibration under the constraint of unchanged total weight or volume. It is intended to investigate the efficiency of this method on space structural.

Structural optimization with vibration involves solution in eigenvalues problems. In general, such optimization is to maximize the fundamental frequency for a given weight or to minimize the weight of structure for a specified frequency. Some methods required scaling of the design variables and others only feasible for two-dimensional modeling. In many space structural modeling, due to the effect of torsional mode, the optimal at the point of increasing the lowest eigenvalue alone may not be satisfactory. The present method takes due consideration for all mode of vibrations and approximately minimizes the fundamental mode of vibration to improve the response of the structure in dynamic excitation.

2. Analytical Method

The present method is based on matrix displacement method and the optimum values are determined by Lagrange Multiplier method. In the present method, the square sum of natural periods of vibration ($\mathbf{T}$) is equivalent to the sum of inversed eigenvalues ($\boldsymbol{\lambda}$). It is obtained by the matrix operation without pre-calculate the eigenvalues[Ref 1]. The objective function takes the form of,

$$\boldsymbol{\Gamma} = \Sigma\,\mathbf{T}^2_k = 4\pi^2\Sigma(1/\boldsymbol{\lambda}_k)$$

$$= 4\pi^2\mathrm{Trace}(\mathbf{S}^{-1}\mathbf{M}) = 4\pi^2\mathrm{Trace}((\mathbf{C}^T\mathbf{D}\mathbf{C})^{-1}\mathbf{M}) \qquad \cdots\cdots\cdots\cdots(1)$$

where $\mathbf{S}$ is the stiffness matrix that later expanded into $\mathbf{C}^T\mathbf{D}\mathbf{C}$. $\mathbf{M}$ is the mass matrix including non-structural masses and k denotes the mode at kth. Matrix $\mathbf{D}$ is a diagonal matrix consists of design variable of cross-sectional areas $\mathbf{A}_i$ and matrix $\mathbf{C}$ is a "connection matrix" associated with the above expansion. To find $\mathbf{C}^{-1}$, a further expansion of matrix $\mathbf{C}$ is necessary due to the existence of non-square matrix for statically indeterminate structures. Furthermore, equation (1) is transformed into an explicit function.[Ref 2]

The objective function is subjected to the constrain of $\Sigma\mathbf{A}_i\mathbf{L}_i = \mathbf{V}$ = constant and side constrains of $\mathbf{A}_i \geq \mathbf{A}_i^1$ and $(l/r)_i \leq (l/r)_i^1$. $\mathbf{A}_i^1$ is the lowest limit of design variables and $(l/r)_i^1$ is the critical slenderness ration of each element. The optimum values are determined by Lagrange Multiplier method. Thus, the Lagrangian function is given as

$$\mathbf{F} = \boldsymbol{\Gamma} + \boldsymbol{\varepsilon}(\Sigma\mathbf{A}_i\mathbf{L}_i - \mathbf{V})$$

and the optimum criterion is now obtained by

$$\partial\mathbf{F}/\partial\mathbf{A}_i = 0 \text{ and } \partial\mathbf{F}/\partial\boldsymbol{\varepsilon} = 0. \qquad \cdots\cdots\cdots\cdots(2)$$

An iteration is necesarry to calculate the optimum cross-sectional area until a prescribed limit of gain in the objective function is reached.

3. Example

To illustrate the efficiency of the present method, a bridge with a single rib is chosen and modeled into space structure (Fig. 1). The bridge has 6 supports at its ends and the non structural masses such as vehicle load is converted into point load directly apply on the node. The results of the analysis are shown in Table 1. It is noted that the $\Gamma_{initial}/\Gamma_{optimum}$ is about 17% and the fundamental frequency has improved about 15%. Although not significant, the second and third modes have improve as well, which shows that all modes have been considered. The changes of cross-sectional areas before and after optimization are illustrated in Fig. 1.

TABLE 1

Analysis	Initial	Optimum
ΣT^2 sec^2	4.849	4.038
T_1	1.213	1.034
T_2	0.895	0.818
T_3	0.765	0.715

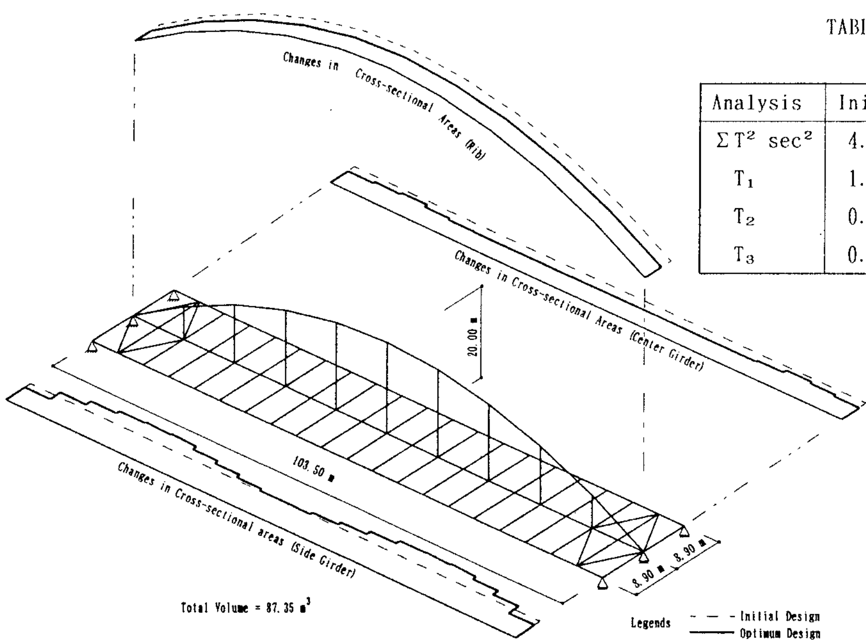

Fig. 1 Single-rib bridge modeled in 3-D

4. Conclusions

The present method is extended to the space structural modeling and its efficiency is investigated. It approximately minimizes the fundamental periods of vibration in a simple manner without pre-calculate the eigenvalues and the rate of convergent is independent of initial values. The present method offers a quick reference for any given structural system to manipulate the natural periods of vibration.

References

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