Fundamental solutions of circular inclined rigid punch on a half plane with an oblique edge crack

Jun Qian, Nagoya Institute of Technology
Norio Hasebe, Nagoya Institute of Technology

INTRODUCTION: A circular punch problem on a half plane with a crack has been studied in the previous paper [1] using complex stress functions. In the present paper, the fundamental solutions of the half plane with an edge crack[2] is used to derive the fundamental solutions of the punch problem on the cracked half plane subjected to concentrated forces. The half plane with an edge crack is first mapped into a unit circle by a rational mapping function so that the forward derivation can be performed on the mapped plane in an analytical way. The fundamental solutions of the half plane with an edge crack is derived by making use of the regularity of the complex stress functions of the half plane. According to the loading and displacement conditions, the punch problem can be transformed into the Riemann-Hilbert problem. To solve the R-H equation, the complex stress functions for the whole problem are divided into two parts, one is the principle part, which is corresponding to the solution of the half plane with an edge crack acted by concentrated forces; the other is the holomorphic part of the problem. By substituting the first part into the R-H equation, and introducing a Plemelj function, the solution of the second part can be obtained explicitly.

THE BOUNDARY CONDITIONS: The problem is shown in Fig.1,which is separated into two parts A and B. Part A is due to the circular punch acted by an eccentric load and subjected to the concentrated forces in the half plane to keep the punch vertical by the moment on the contact region; part B is due to a flat-ended punch inclined with an angle ε by the moment on the contact region without any loads. The half plane with the oblique crack is mapped into a unit circle by a rational mapping function[1].

The loading and displacement conditions for each part can be presented as follows, respectively:

for part A,

$p_x = p_y = 0$ on $L = L_1 + L_2$

$p_x = \mu p_y, \int p_y\, ds = P$ on M

$V_A = x^2 / 2R$ on M

The condition related to the concentrated forces is

$Q(x,y) = (q_x + iq_y)\delta(z,z_0) - (q_x + iq_y)\delta(z,z_m)$

for part B,

$p_x = p_y = 0$ on $L = L_1 + L_2$

$p_x = \mu p_y, \int p_y\, ds = 0$ on M

$V_B = -\varepsilon x$ on M

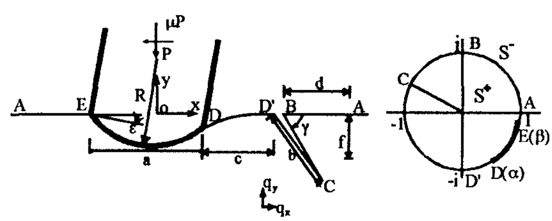

Figure 1 The punch and the unit circle

where $L_1 = ABCD'D$, $L_2 = EA$, $M = DE$ in Fig.1; μ represents the Coulomb's frictional coefficient on M; q_x and q_y represent the components of traction in x and y directions on the surface of the half plane; $Q(x,y)$ represents the concentrated forces in the half plane; $\delta(z,z_0) = 1$ when $z = z_0$ and 0 when $z \neq z_0$, so does $\delta(z,z_m)$.

THE FUNDAMENTAL SOLUTIONS: According to the above loading and displacement conditions, each part can be transformed into the Riemann-Hilbert problem as follows:

$\phi_j^+(\sigma) - \phi_j^-(\sigma) = f_{Lj}$ on $L = L_1 + L_2$

$\phi_j^+(\sigma) + \frac{1}{g}\phi_j^-(\sigma) = f_{Mj}$ on M

where f_{Lj} and f_{Mj} are known functions[1], j = A, B represent parts A and B, respectively.

For part A, the complex stress functions to be obtained are represented by two terms:

$$\phi_A(\zeta) = \phi_{A1}(\zeta) + \phi_{A2}(\zeta)$$
$$\psi_A(\zeta) = \psi_{A1}(\zeta) + \psi_{A2}(\zeta)$$

where $\phi_{A1}(\zeta)$ *and* $\psi_{A1}(\zeta)$ *are the complex stress functions of the half plane acted by the concentrated forces[2].* $\phi_{A2}(\zeta)$ and $\psi_{A2}(\zeta)$ are the holomorphic parts of $\phi_A(\zeta)$ and $\psi_A(\zeta)$, respectively.

By introducing the Plemelj function $\chi(\zeta)$, the general solution of $\phi_{A2}(\zeta)$ can be expressed as

$$\phi_{A2}(\zeta) = H_{A1}(\zeta) + H_{A2}(\zeta) + H_{A3}(\zeta) + \frac{1+i\mu}{2} J_A(\zeta) + Q_A(\zeta)\chi(\zeta)$$

where H_{A1}, H_{A2}, J_A and Q_A can be found from [1], and

$$H_{A3}(\zeta) = \frac{1-i\mu}{2}\frac{1}{2\pi}\left[(\bar{q} - \kappa q)F_1 + (\kappa\bar{q} - q)F_2 + qG_1 + \bar{q}G_2 + 2\pi G_3\right]$$

where F_1, F_2, G_1, G_2 *and* G_3 *are known functions, and* $q = -(q_x + iq_y)/(1+\kappa)$.

For part B, according to the conditions described before, it is obtained that

$$\phi_B(\zeta) = H_B(\zeta) + \frac{1+i\mu}{2} J_B(\zeta) + Q_B(\zeta)\chi(\zeta)$$

where H_B, J_B and Q_B can be found from [1].

THE INCLINED ANGLE OF THE PUNCH: The inclined angle of the punch is determined by

$$R_{mA} + R_{mB} = 0$$

where R_{mA} and R_{mB} denote the resultant moments on the contact region of parts A and B, respectively.

Fig.2 shows the inclined angle of the punch when the concentrated forces act in the body of the half plane. The oblique angle γ of the crack is taken as $60°$, and $b/a = 0.5$, $c/a = 0.0$, $d = a$, $\mu = 0.5$, $\kappa = 2$ and $Ga^2/(PR) = 1$.

THE STRESS INTENSITY FACTORS: Fig.3 shows the non-dimensional stres intensity factors F_I and F_{II} [1] with the concentrated forces acted on the surface of the half plane. $\gamma = 90°$, $b/a = 0.5$, $c/a = 0.0$, $f/a = 0$, $\mu = 0.5$, $\kappa = 2$ and $Ga^2/(PR) = 1$.

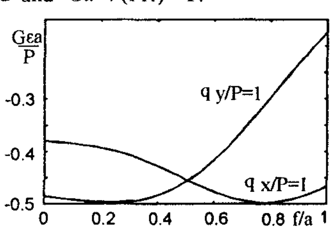

Fig.2 The inclined angle of the punch

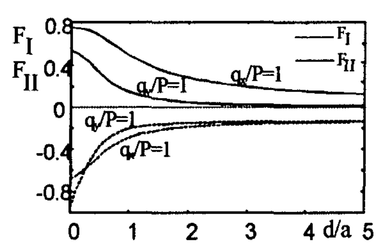

Fig. 3 The non-dimensional stress intensity factors

CONCLUSIONS: The fundamental solution obtained in the present paper can be progressively used to form special type of boundary element. It has certain advantage since the boundary conditions are completely satisfied.

REFERENCES

1. Hasebe, N. & Qian, J. Circular inclined punch problem with two corners to contact with a half plane with a surface crack , *Contact Mechanics II*, Ed. Aliabadi M. H. and Alessandri C., Computational Mechanics Publications, 1995, Southampton, 159-166.
2. Hasebe, N., Qian, J. & Chen, Y.Z. Fundamental solutions for half plane with an oblique edge crack (submit to the 11th International Conference of BEM)