A Constitutive Model of an Interface and its Application

Jian-Guo WANG Nagoya University Student member
Yasuaki ICHIKAWA Nagoya University Member

1 Introduction

Rock mass is typical fissured material, its macro-properties strongly depend on the geometrical and the physical properties of the interfaces. Authors have shown this dependence in another paper[1], where it is assumed that the interface is linear response. In this paper, we will show: (1) a constitutive model of an interface which takes the dilatancy, stress/strain history into account; (2) performance of this constitutive model; (3) its effect on the fissured rock massses by means of homogenization theory[2].

2 Proposed constitutive model of an interface

2.1 The constitutive relation of layer material

$$\{d\sigma\} = ([D] - [D_p])\{d\varepsilon\} \qquad [D_p] = \frac{H(l)}{A}[D]\{\frac{\partial g}{\partial \sigma}\}\{\frac{\partial f}{\partial \sigma}\}^T[D]^T \tag{1}$$

$$A = A^* + \{\frac{\partial f}{\partial \sigma}\}^T[D]\{\frac{\partial g}{\partial \sigma}\} \qquad A^* = (-)\frac{\partial f}{\partial H}(\frac{\partial g}{\partial \sigma_y}\frac{\partial H}{\partial \varepsilon_v^p} + \frac{\partial g}{\partial \tau_{xy}}\frac{\partial H}{\partial \bar{\varepsilon}^p}) \tag{2}$$

where f is the yield function with hardening function $H(\varepsilon_v^p, \bar{\varepsilon}^p)$; g is the potential function; and $[D]$ is elastic matrix of materials. ε_v^p is plastic volumetric strain; $\bar{\varepsilon}^p$ is generalized shear plastic strain. $H(l)$ is Heaviside function and $l = \{\frac{\partial g}{\partial \sigma}\}^T\{d\sigma\}$. $f = g$ if associated flow rule is adopted. The failure criterion is $A^* = 0$

$$\begin{cases} l > 0 & H(l) = 1 & \text{:elastoplastic deformation} \\ l \le 0 & H(l) = 0 & \text{:elastic deformation} \end{cases}$$

2.2 A constitutive model of a regular interface

A regular interface can be regarded as a limit of layer material zone as its thickness of this zone trends to zero (see Fig.1). That is:

$$[[\![du]\!]_J] = [[\![du_s]\!],\ [\![du_n]\!]]^T = \lim_{b\to 0}[b\gamma_{sn},\ b\varepsilon_n]^T \tag{3}$$

Define $K_n = \frac{1}{b}(K + \frac{4}{3}G)$, $K_s = \frac{G}{b}$ and $\{d\sigma\} = \{d\sigma_n\ d\tau_{sn}\}^T$. From Eq(1) it is easily to get

$$[\bar{D}] = \begin{bmatrix} K_n & 0 \\ 0 & K_s \end{bmatrix} \qquad [\bar{D}_p] = \frac{1}{\bar{A}}[\bar{D}]\{\frac{\partial g}{\partial \sigma}\}\{\frac{\partial f}{\partial \sigma}\}^T[\bar{D}]^T$$

$$\bar{A} = \bar{A}^* + \{\frac{\partial f}{\partial \sigma}\}^T[\bar{D}]\{\frac{\partial g}{\partial \sigma}\} \qquad \bar{A}^* = (-)\frac{\partial f}{\partial H}(\frac{\partial g}{\partial \sigma_n}\frac{\partial H}{\partial u_n} + \frac{\partial g}{\partial \tau_{sn}}\frac{\partial H}{\partial u_s})$$

The particular form of yield function

$$f = g = \frac{(\sigma_n - H)^2}{C} + \frac{\tau_{sn}^2}{B} - H = 0 \quad \text{and} \quad H = H(h) = H(m_1 + m_2u_n + m_3u_s) \tag{4}$$

thus the stress–strain relation is

$$\left\{\begin{array}{c}\Delta\sigma_n\\ \\ \Delta\tau_{sn}\end{array}\right\}=\left[\begin{array}{cc}K_n-\frac{H(l)}{\bar{A}}\bullet\frac{4}{C^2}K_n^2(\sigma_n-H)^2 & -\frac{H(l)}{\bar{A}}\bullet\frac{4K_sK_n}{BC}(\sigma_n-H)\tau_{sn}\\ -\frac{H(l)}{\bar{A}}\bullet\frac{4K_sK_n}{BC}(\sigma_n-H)\tau_{sn} & K_s-\frac{H(l)}{\bar{A}}\bullet\frac{4K_s^2}{B^2}\tau_{sn}^2\end{array}\right]\left\{\begin{array}{c}[\![\Delta u_n]\!]\\ \\ [\![\Delta u_s]\!]\end{array}\right\} \quad (5)$$

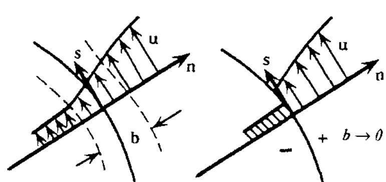
$$\bar{A}=\frac{4}{C^2}(\sigma_n-H)^2K_n+\frac{4}{B^2}\tau_{sn}^2K_s+\bar{A}^* \quad \bar{A}^*=\frac{4}{C}[\sigma_n+(C-1)H][m_2\frac{\sigma_n-H}{C}+m_3\frac{\tau_{sn}}{B}]\frac{dH}{dh}$$

where u_n or $[\![u_n]\!]$ is the normal displacement and u_s or $[\![u_s]\!]$ is the shear displacement[3]. Eq(5) is expressed in local coordinates $\mathbf{n}-\mathrm{s}$. The failure criterion $\bar{A}^*=0$ implies that

$$\frac{\tau_{sn}}{\sigma_n}=\text{Constant} \quad (6)$$

3 Performance of this constitutive model

Fig.2 is the performance of one interface. This constitutive model is introduced into the homogenization theory proposed by authors[1]. Its effect on the macro-mechanical properties of fissured rock cell will be shown in the presentation.



Continuous Zone   Discontinuous Interface

Fig.1 Idealization of Discontinuous Interface

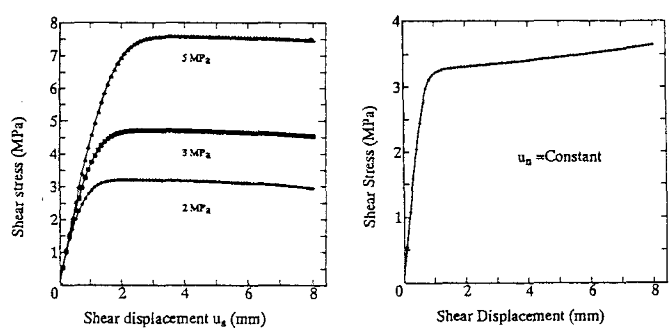


Fig.2 Performance of the constitutive model of interfaces

Conclusion

The limit concept is possible to be applied to constitutive law of interfaces. The proposed elastoplastic constitutive model can describe the nonlinearity, dilatancy and friction. Slipping is the most important factor to the macro-mechanical properties of fissured rock.

References

[1] Jian-Guo WANG and Yasuaki ICHIKAWA(1994), A homogenized constitutive law with distributed discontinuities in rock mass, to be submitted. [2] Jian-Guo WANG, Kohichi TSUJI and Yasuaki ICHIKAWA(1994), Evolution of micro-stress and macro-mechanical behaviors of nonlinear geomaterials, to be published in Proceedings of the Japan Congress on Materials Research [3] Lanru JING(1990), Numerical modelling of jointed rock masses by distinct element method for two, and three-dimensional problems, PhD Thesis, Lulea University of Technology