IMPROVEMENT ELASTO−PLASTIC ANALYSIS OF STEEL SHELLS

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1. INTRODUCTION

This paper presents an elasto−plastic finite displacement analysis of shell structures. The analytical method is based on the degenerated isoparametric shell element in three dimensional space. The analytical program, which had been developed in this study, considers material nonlinearity due to elasto−plastic behavior as well as geometric nonlinearity due to large displacement of structures [1]. Von Mises yield criterion and Prandtl−Reuss flow rule are adopted for material nonlinearity. An arc−length method is employed on this analytical program, especially to carry out snap through or snap back problems. Numerical examples are demonstrated.

2. ANALYTICAL METHOD

A procedure of elasto−plastic analysis is based on the method which had been developed in [3]. The elasto−plastic behavior of steel with strain hardening is considered in the same manner of Ref. [4]. The elasto−plastic formulation for degenerated shell element is derived as below. An elasto−plastic matrix can be written as,

$$D_{EP} = D_E - \frac{D_E\{\partial f/\partial \boldsymbol{\sigma}\}\{\partial f/\partial \boldsymbol{\sigma}\}^T D_E}{\{\partial f/\partial \boldsymbol{\sigma}\}^T D_E\{\partial f/\partial \boldsymbol{\sigma}\}+H'} \quad (1)$$

which D_E, σ_V, ε_P are elasticity matrix, effective stress, effective plastic strain, respectively.

$$\boldsymbol{\sigma} = \{\sigma_x \ \ \sigma_y \ \ \tau_{xy} \ \ \tau_{yz} \ \ \tau_{xz}\} \quad (2)$$

$$f = (\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2 + 3\tau_{yz}^2 + 3\tau_{xz}^2)^{1/2} = \sigma_V = k \quad (3)$$

$$H' = dk/d\varepsilon_P \quad (4)$$

Generally, stress−strain relationship of steel material has three states, elastic, perfectly plastic, and plastic with hardening, as shown in Fig. 1. Concerning equation (1), these three states can be explained as follows,

1. $\sigma_V < \sigma_Y$ where $D = D_E$ (5)
2. $\sigma_V \geqq \sigma_Y$ and $\varepsilon_V < \varepsilon_P$ where $D = D_{EP} \equiv D_P$ $(H' = 0)$ (6)
3. $\sigma_V \geqq \sigma_Y$ and $\varepsilon_V \geqq \varepsilon_P$ where $D = D_{EP} \equiv D_H$ $(H' \neq 0)$ (7)

Then, there are two transition states i.e. P → R and S → U. For these transition states, stress−strain matrix is obtained as follow,

$$D = \alpha D_E + (1-\alpha) D_P \qquad (P \rightarrow R) \quad (8)$$

$$D = \beta D_P + (1-\beta) D_H \qquad (S \rightarrow U) \quad (9)$$

The magnitude of α can be derived as the following manner. When stress level is at elastic state which is shown by point P in Fig. 1, we have

$$\sigma_V^2 = [\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2 + 3\tau_{yz}^2 + 3\tau_{xz}^2] < \sigma_Y^2 \quad (10)$$

After incremental stress $\triangle\sigma$, the effective stress becomes greater than σ_Y. Multiplying $\triangle\sigma$ with α and rearanging, we have

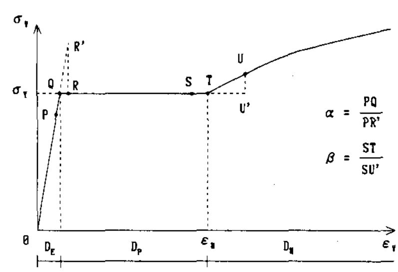

Fig. 1 Stress strain diagram

$$\sigma_Y^2 = (\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2 + 3\tau_{yz}^2 + 3\tau_{xz}^2) + (2\sigma_x\triangle\sigma_x - \sigma_x\triangle\sigma_y - \sigma_y\triangle\sigma_x + 2\sigma_y\triangle\sigma_y + 6\tau_{xy}\triangle\tau_{xy} + 6\tau_{yz}\triangle\tau_{yz} + 6\tau_{xz}\triangle\tau_{xy})\alpha + (\triangle\sigma_x^2 - \triangle\sigma_x\triangle\sigma_y + \triangle\sigma_y^2 + 3\triangle\tau_{xy}^2 + 3\triangle\tau_{yz}^2 + 3\triangle\tau_{xz}^2)\alpha^2 \quad (11)$$

Now the above equation can be constructed as quadratic equation,

$$A\alpha^2 + B\alpha + C = 0 \quad (12)$$

where

$$A = \triangle\sigma_x^2 - \triangle\sigma_x\triangle\sigma_y + \triangle\sigma_y^2 + 3\triangle\tau_{xy}^2 + 3\triangle\tau_{yz}^2 + 3\triangle\tau_{xz}^2 \quad (13)$$

$$B = 2\sigma_x\triangle\sigma_x - \sigma_x\triangle\sigma_y - \sigma_y\triangle\sigma_x + 2\sigma_y\triangle\sigma_y + 6\tau_{xy}\triangle\tau_{xy} + 6\tau_{yz}\triangle\tau_{yz} + 6\tau_{xz}\triangle\tau_{xy} \quad (14)$$

$$C = \sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2 + 3\tau_{yz}^2 + 3\tau_{xz}^2 - \sigma_y^2 = \sigma_V^2 - \sigma_Y^2 \quad (15)$$

$$\alpha_{1,2} = \frac{-B \pm \sqrt{(B^2-4AC)}}{2A} \quad (16)$$

3. Numerical Examples

3.1 Shallow Cylindrical Shell

A quarter of cylindrical shell is modeled by 3x3 8−node Serendipity elements. Vertical load is applied at the center of the shell as shown in Fig. 2. This problem which have snap back behavior was investigated by several reserchers. The present results as shown in Fig. 3 is in good agreement as compared with Ref. [2].

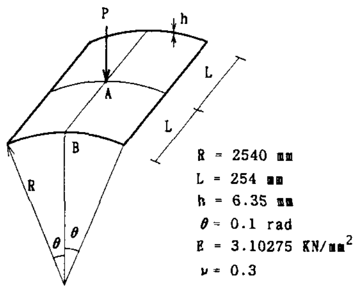

Fig. 2 Shallow cylindrical shell

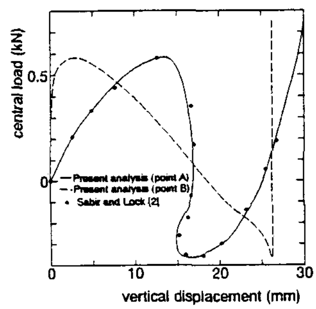

Fig. 3 Load deflection curve

3.2 Imperfect Square Plate

For the imperfect square plate, 3x3 elements mesh was used to analyze a quarter of the plate. Uniform axial displacement or uniform membrane load is applied at X=0, as shown in Fig. 4. Initial deflection is expressed by the following equation,

$$Z(X,Y) = w_0 \sin\left(\frac{\pi X}{a}\right)\sin\left(\frac{\pi Y}{b}\right) \tag{17}$$

in which w_0 is initial deflection at the center of the plate. Good agreements are attained both for nonlinear elastic solution and elasto−plastic solution, as shown in Fig. 5.

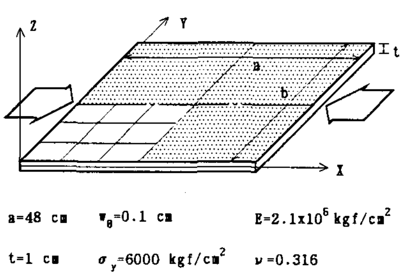

Fig. 4 Imperfect square plate

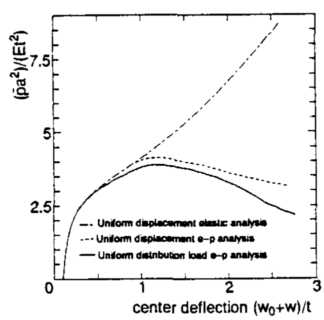

Fig. 5 Load deflection curve

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