Thin Plate Bending Problem of Partially Bonded Bimaterial Strips

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Introduction

A problem of considerable practical importance is that, of bending of two strips with different rigidities and elastic properties, partially bonded along a finite straight line. The problem represent idealization of the bending of two dissimilar materials, such as; welded materials and bonded metallic and nonmetallic materials. Complex stress function and rational mapping function are utilized to obtain the general solution. The problem is analyzed for a concentrated bending moment applied at the tip of the strips. Values of stress intensity of debonding (SID), at the debonding tips are given in demonstrative graphs.

Stress Intensity of Debonding

Fig. 1(a) illustrates the physical plane of a strip composed of two dissimilar materials bonded symmetrically with respect to the interface, while Fig. 1(b) exhibits the unit circles of the mapped planes. Since the derivation of the complex stress function $\phi_j(t_j)$ was previously reported in [1], this paper dedicates the derivation of the SID at the debonding points C and D of Fig. 1. The first derivative of the complex stress function $\phi_j(t_j)$ can be expressed as follows:

$$\phi'_j(t_j)=\frac{\chi_j(t_j)}{(t_j-\alpha)(t_j-\beta)}F_j(t_j)+\overset{0}{F}_j(t_j) \quad ; j=1,\ 2 \quad \ldots\ldots\ldots\ldots (1)$$

where; $\chi_j(t_j)$ is Plemelj's function defined by,

$\chi_j(t_j)=(t_j-\alpha)^{m_j}(t_j-\beta)^{1-m_j}$; $m_j=0.5+i\varepsilon_j$; $\varepsilon_j=(\ln\lambda_j)/(2\pi)$.

where ε_j and λ_j are constants expressed in terms of rigidities D_j and Poisson's ratios ν_j of the two materials.

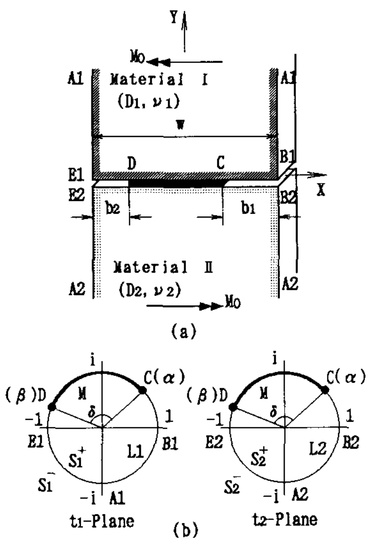

Fig.1. (a) Physical Plane; (b) Unit Circles

The first derivative of the complex stress function in the physical plane can be expressed in terms of the SID at the debonding points C and D, respectively, as follows:

$$\Phi'_j(z_j)=\frac{K_c^{(j)}\exp(\pi\varepsilon_j)}{2\sqrt{2\pi}}(z_j-z_c)^{m_j-1}+O(r^0) \quad ; \quad \Phi'_j(z_j)=\frac{K_d^{(j)}\exp(\pi\varepsilon_j)}{2\sqrt{2\pi}}(z_j-z_d)^{-m_j}+O(r^0) \quad \ldots\ldots (2)$$

where $K_c^{(1)}=\frac{c_2}{c_1}\overline{K_c^{(2)}}\equiv K_c$; $K_d^{(1)}=\frac{c_2}{c_1}\overline{K_d^{(2)}}\equiv K_d$; $c_j=k_jD_j(1-\nu_j)$; $k_j=(3+\nu_j)/(1-\nu_j)$.

The terms $\sqrt{K_c\overline{K_c}}$, $\sqrt{K_d\overline{K_d}}$ represent the SID at points C , D respectively, and they are obtained by the following expressions:.

$$\frac{K_c^{(j)}\exp(\pi\varepsilon_j)}{2\sqrt{2\pi}}=\frac{|\omega'(\alpha)(\alpha-\beta)|^{1-m_j}}{\omega'(\alpha)(\alpha-\beta)}F_j(\alpha)\exp[i(1-m_j)(\theta_b-\theta_a)] \quad \ldots\ldots\ldots\ldots\ldots\ldots (3a)$$

$$\frac{K_d^{(j)}exp(\pi \varepsilon_j)}{2\sqrt{2\pi}}=\frac{|\omega'(\beta)(\beta-\alpha)|^{m_j}}{\omega'(\beta)(\beta-\alpha)}F_j(\beta)\ exp[-im_j(\theta_b-\theta_a)] \quad \text{(3b)}$$

where, $\theta_b-\theta_a=\pi+\delta/2$; δ is the central angle on the unit circle between points α and β, shown in Fig.1(b). The following dimensionless SID is used, which has a finite value:

$$F=D_j(1-\nu_j)\sqrt{K^{(j)}\overline{K^{(j)}}}\ cosh(\pi \varepsilon_j)\frac{\sqrt{2\{w-(b_1+b_2)\}}}{M_0\sqrt{\pi}} \quad \text{(4)}$$

Another expression which demonstrates the physical meaning of the SID is used as follows:

$$N=D_j(1-\nu_j)\sqrt{K^{(j)}\overline{K^{(j)}}}\ cosh(\pi \varepsilon_j)\frac{\sqrt{2w}}{M_0\sqrt{\pi}} \quad \text{(5)}$$

Results and Discussion

In order to examine the actual phenomena at the deponding points, two expressions has been deduced, as given by eqs.4 & 5. These values are shown in Figs.2 & 3, which show the values for b1/w for some b2/w for rigidity ratio $D_2/D_1=0.5$; $\nu_1=0.5$; $\nu_2=0.25$. The solid lines give the values of SID at point C, and the dashed lines give those of point D, while the dotted line which delineates a value of 0.6378 is a demonstration for the value of the SID when the length of the bond line CD of Fig.1 becomes very small with respect to the strip width. The problem turns to be a model for that of bending of partially bonded semi-infinite planes, for which the values of SID are also obtained by interchanging the mapping function in the foregoing formulations. The values of the SID in Fig.3 illustrate the physical values, from which it is obvious that the side whose debonding length is longer has a larger N value. Hence its rate of growing is rapid. The values increase monotonically and so the debonding develops until the fracture occurs

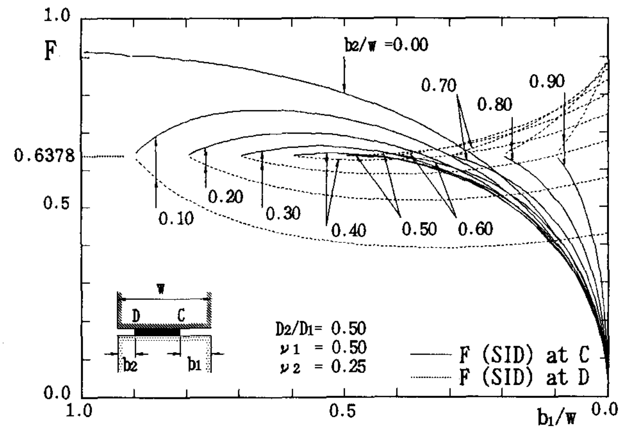

FIG. 2. Dimensionless Stress Intensity of Debonding at the Debonding Tips C and D.

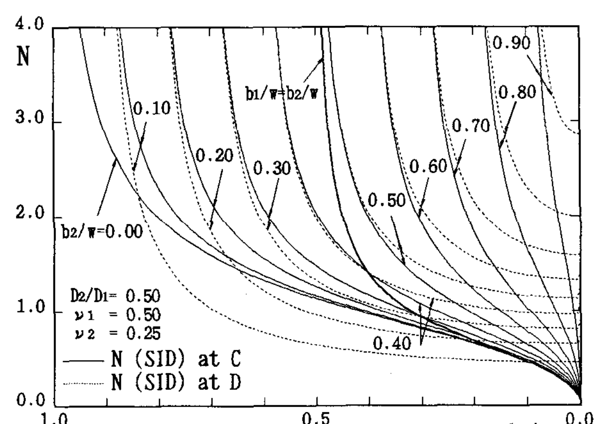

FIG. 3. Dimensionless Stress Intensity of Debonding at the Debonding Tips C and D. (Physical Meaning)

Referrances

1)Salama M. et al. National Congress for Applied Mechanics, 1993