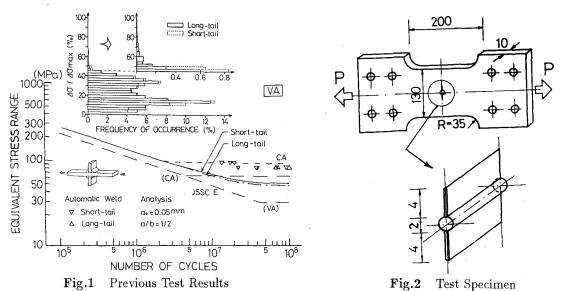
# Analytical Model of Fatigue Life under Overloading Condition

Nagoya University

O Xiaohua CHENG Kentaro YAMADA

## 1.Introduction

It is well known that a single overload or multi-overload can reduce crack growth rate significantly at the lower stress range level following the overloads. The previous fatigue test result of steel welded joints shows that the fatigue life under variable amplitude spectrum loading with long tail in higher load region is longer than the fatigue life under short tail spectrum loading. The comparison between fatigue test data and fatigue life analytical result also shows that at lower stress level the fatigue test life under long tail spectrum loading is much higher than computed life (Fig.1). It is believed that the retardation effect due to overload should be considered in the analysis.



But it seems that the quantitative studies on retardation effect with consideration of crack closure phenomenon have not yet been carried out to structural steel. In order to investigate the retardation effect well, an analytical model is developed and overload fatigue test is being carried out to verify the analytical model. The concept of opening stress or effective stress intensity factor range is introduced into empirical formula of crack growth rate to evaluate the delayed fatigue life of welded joints. The center cracked fatigue test specimen is shown in Fig.2.

#### 2. Fracture Mechanics Approach

The empirical formula of crack growth rate  $da/dN = C(\Delta K^m - \Delta K_{th}^m)$  is generally used to calculate fatigue life under constant amplitude and variable amplitude loading. However, because of the existence of larger plastic zone near crack tip due to overload, the crack growth rate is retarded after overload due to the crack-closure phenomenon, which is caused by residual plastic deformation remaining in the wake of an advancing crack tip (Fig.3(b)). In order to start the crack opening at this time, the opening stress level  $\sigma_{op}$  is needed. The opening stress can be calculated from contact stress  $\sigma_c(x)$  due to crack closure.

$$\sigma_{op} = \sigma_{min} + \frac{2}{\pi} \left( \int_0^a \frac{\sigma_{c}(x)}{\sqrt{a^2 - x^2}} F\left(\frac{a}{w}\right) dx \right)$$
 (1)

where  $F(\frac{a}{m})$  is correction factor of finite plate width and others are shown in Fig.3.

In an other word, the effective stress intensity factor range  $\Delta K_{eff}$  (Eq.(2)) becomes lower which results in the lower fatigue crack growth rate (Fig.3(a)). The correction coefficient of effective stress intensity factor U is used to show the retardation effect on crack growth, quantitatively.

$$\Delta K_{eff} = U \Delta K = \left(\frac{\sigma_{max} - \sigma_{op}}{\sigma_{max} - \sigma_{min}}\right) \Delta K \tag{2}$$

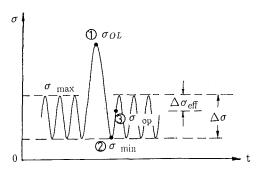
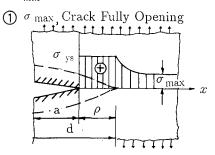


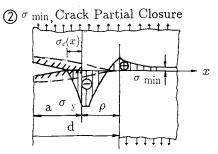
Fig.3(a) Stress Sequence

As effective stress intensity factor range  $\Delta K_{eff}$  for each load cycle following overload can replace the value of  $\Delta K$  without overload effect, the empirical formula of crack growth rate can be represented by  $da/dN = C(\Delta K_{eff}^m - \Delta K_{th}^m)$  to calculate fatigue life under overloading condition.

#### 3. Stress Distribution Analysis

First, the elasto-plastic FEM analysis is carried out to investigate the stress state of the specimen. As the stress in direction of plate thickness is relatively small, the stress state is assumed to be planar. In order to determine the contact stress  $\sigma_c(x)$  due to crack closure, numerical computation is carried out in element method. The element pattern for the numerical analysis is the bar element within Dugdale's plastic zone.





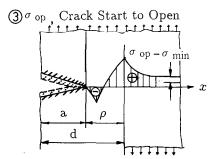


Fig.3(b) Crack Opening-Closure Model

## 4. Analytical Results on Fatigue Crack Growth Rate and Fatigue Life

From above stress analysis, the contact stress distribution can be obtained. The crack opening stress  $\sigma_{op}$  can be calculated from Eq.(1) and hence  $\Delta K_{eff}$  from Eq.(2). The analytical result by using this model will be compared with test results and previous analytical results without considering the retardation effect. The results will be presented later.

### Reference

J. C. Newman. Jr., "A Crack-Closure Model for Predicting Fatigue Crack Growth under Aircraft Spectrum Loading", ASTM STP 748, Oct. 1981