BENEFIT ESTIMATION OF NEWLY TRANSPORT FACILITY

*Hisayoshi Morisugi and *Le Dam Hanh*
Department of Civil Engineering, Gifu University

1. INTRODUCTION

The previous studies mainly pay attention to evaluate the improvement of existing facilities. There are, however, no concrete method for the newly introduced transport facility case. In the case of newly introduced facility we have to cope with the problem of estimating the value of *cost of new transport mode that is considered corresponding with "without project" situation (where the new mode* dose not exist yet), which seems to be difficult. Therefore, we propose to look at the inverted demand function instead of demand function itself in the estimation of the change in household's benefit by the newly introduced transport mode, by using the high-order of Taylor expansion.

2. BENEFIT MEASUREMENT

Based on household's utility maximizing approach, let z be a vector of consumer goods; y is income; p, x are price and demand of newly introduced transport service, respectively. Then let the individual maximizes his utility by controlling z and x under his budget constraint:

$$V(p,y) \equiv U(z(p,y), x(p,y)) = \max_{z,x} \{U(z,x) \mid z+px=y\}$$

where, U(.) and V(.) the direct and indirect utility functions, respectively. Now we assume that given the level of x at x(p,y) then the problem is only optimization of z which is considered as follows:

$$\max_{z} \{U(z, x(p,y)) \mid z=y-px(p,y)\}$$

Then

$$\max U = V(x(p,y), y-p.x(p,y)) = V(p,y)$$

Since in the "without project" situation where the new facility is not yet exist, obviously it demand is equal to zero as: w/o : x(.) = 0

Suppose that, the new facility is introduced with a given demand function say,

w : x = x(p,y)

The newly introduced mode do provide benefits to users that can be measured by using the concept of equivalent variation, EV, that is defined as the minimum amount of compensation which is needed for an individual in order to give up the project while sustaining his welfare level at the " with project" situation. It can be defined *formally as:*

$$V(p^0, y^0 + EV) = V(p^1, y^0) \qquad (1)$$

where, 0 and 1 denote two states (w/o and w project). Since the expenditure function e(P,U), (where U denotes direct utility function) is simply the inverse of V(P,Y) in y, (1) can easily be shown that

$$EV = \{e(P^0, V^1) - e(P^0, V^0)\} \qquad (2)$$

where, $V^1 = V(P^1, Y^0)$, $V^0 = V(P^0, Y^0)$

In the case of new transport mode, however, the problem is how to measure value of p^0. In order to overcome this problem, we are reforming EV as function of demand, x, directly by *looking at the inverted demand* function, say, p which is a function of x. EV can be transformed as :

$$EV = \{e(x=0,\ V(x=x(p,y),\ y-px(p,y))\} - \{e(x=0, V(x=0,y)\} \qquad (3)$$

Because

V(x=x(p,y), y−px(p,y)= V(p,y), and V(x=0,y) correspond with indirect utility level of "with project" and "without project" situation, respectively.

(3) can be rewritten as:

$$\begin{aligned} EV = {} & e(x=0, V(x=x(p,y),\ y-px(p,y)) \\ & - e(x=0,\ V(x=x(p,y),y) \\ & + e(x=0,\ v(x=x(p,y),y)) \\ & - e(x=0,\ V(x=0,Y)) \end{aligned} \qquad (4)$$

Denoting that, the first, second and forth terms of right hand side of equation (4) as (I), (II), (III) respectively, then

(I) is the value of e(.) for :

$$V(x=o, e(.)) = V(x=x(p,y), y-px(p,y))$$

(II) is the value of e(.) for :

$$V(x=0,\ e(.)) = V(x=x(p,y),y)$$

(III) is the value of e(.) for:

$V(x=0,\ e(.)) = V(x=0, y)$

Setting :

$EV_1 = (I)-(II)$ and $EV_2 = (II) - (III)$

$EV = EV_1 + EV_2$

Then, EV_1 can be transformed into Integral form yields :

$$EV_1 = \int_y^{y-px} e_v\ v_y\ dy = -px$$

where, the subscript indicates partial derivatives with to the supscripted valuable

$EV2 = e(0, V(x, y)) - e(x, V(x, y))$ (a)

$+ e(x, V(x, y)) - e(x, V(0, y))$ (b)

$+ e(x, V(0, y)) - e(0, V(o, y))$ (c)

By turn, transforming (a), (b) and (c) in to Integral forms, then

$$(a)+(c) = \int_x^0 [P^c(x, V^0) - P^c(x, V^1)]dx$$

where, $P^c(x, V)$ is the inverse function of compensated demand function for the fixed level of V. On the other hand

$(b) = -\{e(x, V(0, y)) - e(x, V(x, y))\}$

$$(b) = -\int_x^0 e_y\ p(x, y)\ dx$$

Next using Taylor expansion around a point x, so (b) can be obtained as follows:

$- e_v\ p(x, y)\Delta x - (1/2)\partial/\partial x[e_v\ p(x, y)]x^2 + \ldots\ldots\ldots$

where, $(\Delta x) = (0-x)$, 0 and x correspond to demand of new transport mode of two situation "without" and "with" project, respectively.

Since we are expanding at x

$e_v\ |_x = 1 \quad e_{vv}\ |_x = 0$

and

$\partial/\partial x[e_v\ p(x, y)]|_x = e_{xv}\ p(x, y) + e_v p_x\ |_x$

$e_{xv}\ |_x = \partial/\partial y[e_v\ p(x, y)]|_x = p_y\ |_x$

Substitute these results into (b):

$(b) = p(x, y)x - 1/2\ (p_x\ |_x)x^2$

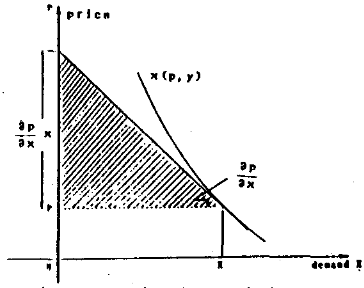
Here, if no income effect is assumed, (i.e, $\partial p/\partial y = 0$) then we can show that $(a)+(c) = 0$

therefore, $EV = EV1 + EV2$

$EV = -(1/2)p_x(x, y)|_x x^2$ (5)

In figure 1. value of EV can be shown as the shaded area.

Now since (5) shows that the expression obtained for EV can be readily measured in more traditional form, it seems easy to calculate with specified demand function.



Application to logit model, that assumes, one existing mode, say mode 1 given a demand $x_1 = 1$. Now the situation is one new mode is introduced with a demand, say x. Therefore, now $x_1 = 1-x$ (by assumption of total demand remain constant), then we can assume our new mode demand function as

$x = 1/\ [1+ \exp(-f(p))]$

and $\partial x/\partial p = x\ (1-x)f_p$

where, f_p denotes partial differentiation of $f(.)$ with respect to p. Substitute the above result into equation (5), our conclusion now become more clearly as

$EV = (1/2)\{x(x-1)\}^{-1}(\partial f/\partial p)^{-1}x^2$

$= (1/2)\ [x/(1-x)]f^{-1}$

Now the problem is only the function form of $f(.)$. Usually, $f(.)$ is assumed to be in linear form as $f = b - ap$, where a, b are parameters, then our EV value is estimated as

$EV = (1/2a)[x/(1-x))]$

3. CONCLUSION

Viewing from the resulting EV, we come with the conclusion that the inverted demand function are useful to overcome the problem of newly introduced facility, but limited by the assumption of no income effect. However, since many urban transportation facility could be assumed no income effects, then our result might be applicable to these facily.

Reference:

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