# Development of A New Two-Surface Model for Steels under Cyclic Loading

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## INTRODUCTION

The mechanical behavior of the material under the cyclic loading is much complicated than that of monotonic loading. Recently, a great attention has been paid to the two-surface model, which is used to predict the behavior of structural steel under cyclic loading. The two-surface model was proposed by Dafalias and Popov[1] in 1975, which is based on the Mróz multi-surface model[2]. Further, the Dafalias model was modified by Peterson[3], Tseng[4], and Cofie[5]. But in many cases, those models can not give a satisfying prediction of the elasto-plastic behavior of structural steel under cyclic loading because an important feature of the yield plateau is not treated well. In this paper, a new two-surface model is proposed.

## THE PROPOSED MODEL

The two-surface model by Dafalis assumes that there exists the bounding line (XX' and YY' in Fig. 1) and that the plastic modulus $E^P$ is related to the distance $\delta$ between the loading point and the bounding line. It can be expressed as follows:

$$E^P = E_0{}^P + h\frac{\delta}{\delta_{in} - \delta} \tag{1}$$

where $E_0{}^P$ is the slope of the bounding line, h is the shape parameter, $\delta_{in}$ is the initial $\delta$ of the current loading path, which is different for every loading loop.

Based on the Dafalias two-surface model, the *following* four parts has been implemented in the proposed model.

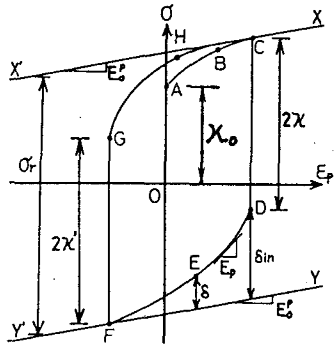


*Fig. 1 The concept of* the bounding line

1. The Reduction of the Elastic Range during *Plastic Deformation*

According to the experiment of the SS41 and SM50 structural specimens, it was found that the elastic range k decreases with the increment of the effective accumulated plastic strain $\bar{\varepsilon}^P$. When $\bar{\varepsilon}^P$ reaches a certain value, k becomes almost constant. The mathematical relationship of the elastic range and the effective accumulated plastic strain was obtained from the experimental data.

$$\kappa/\kappa_0 = \alpha - a\mathrm{EXP}(-b\bar{\varepsilon}^P \times 100) - (\alpha - a - 1)\mathrm{EXP}(-c\bar{\varepsilon}^P \times 100) \tag{2}$$

where $\kappa_0$ is the initial elastic range(shown in Fig. 1); $\alpha$, a, b and c are the material parameters; $\bar{\varepsilon}^P$ means the deffence between the maximum and minimum plastic strain that the material has experienced.

2. Modification of Shape Parameter h

The shape parameter h in Eq.(1) dominates the curvature of the stress-strain curve. In the Dafalias model, it was considered as a constant during one loading cycle. Actually, it was observed that the parameter h changes with $\delta$ during the plastic deformation. For simplicity, the linear relation between h and $\delta$ was adopted in the model.

$$h = d\delta + e \tag{3}$$

where d and e depend on the material.

3. Introduction of the Concept of Virtural Bounding Line

An important phenomena has been found in the experiment, that is when an unloading point does not lie on the bounding line(see point A in Fig. 1), the behavior of next reloading history can not be predicted well by the existed two-surface model. In the present model, the concept of the virtural bounding line is proposed(as shown in Fig. 2). It is assumed that the virtural bounding line for the current loading path is parallel to the original bounding line and passes the latest unloading point. When the loading point reaches the original bounding line, it behaves as the linear hardening.

4. Treatment of the Yield Plateau

The length of the yield plateau is usually considered as constant for the loading. But, the experimental evidences of the SS41 and SM50 specimens show that the yield plateau length becomes shorter after some plastic loading loops. The relationship between the plastic work and $\bar{\varepsilon}^P$ at the terminal point of the yield plateau is assumed to be as shown in Fig. 3.

$$\frac{\varepsilon^P}{\varepsilon^P_{ST}} - 1 = M\left(\frac{W^P}{W^P_{ST}} - 1\right) \qquad (4)$$

where $\bar{\varepsilon}^P$ is the same as in Eq.(2); $\varepsilon^P_{ST}$ and $W^P_{ST}$ indicate the plastic strain and plastic work at the end of the yield plateau under the monotonic loading and M is a constant, which is shown in Fig. 3.

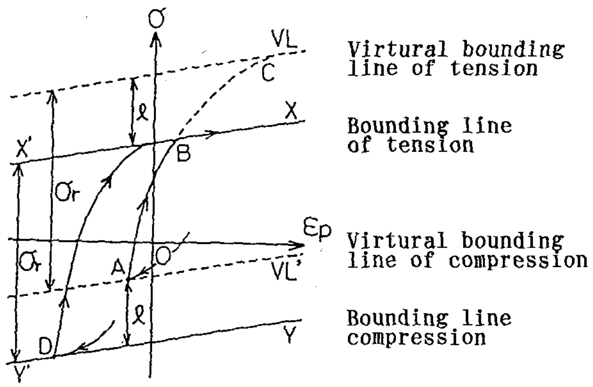


Fig. 2 The concept of virtual bounding line

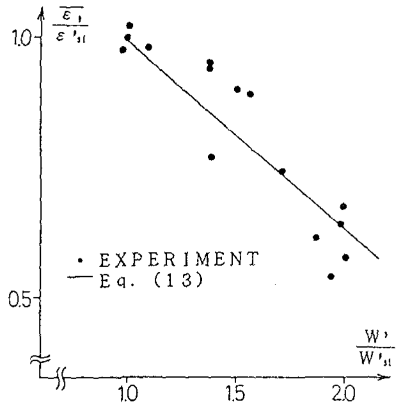


Fig. 3 Reducing of the yield plateau

## THE EXAMPLES

Figures 4 and 5 are the examples of prediction of the SS41 and SM50 specimens under cyclic loading respectively. They showed a good agreement between the experiment and the prediction by the proposed model. For the SS41 material, the plastic strain of the yield plateau is 1.53%, it reduced to 1.08% in this example because of the plastic work.

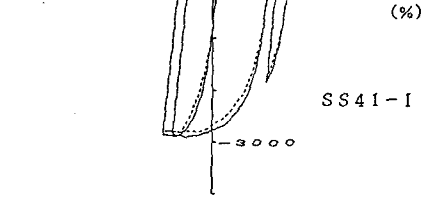


Fig. 4 Prediction for SS41

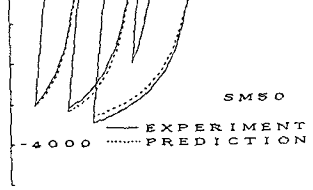


Fig. 5 Prediction for SM50

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