AN ELASTO-PLASTIC LARGE DEFORMATION ANALYSIS OF SHELLS INCLUDING LIMIT POINTS

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1.INTRODUCTION

A geometrically and materialy nonlinear displacement based shell finite element is presented in this paper.Although the main purpose of this study is to analyze axially loaded cylindrical shells with initial imperfections,the element was developed so that it can be used for the analyses of beams,plates and shells using displacement increment method as well as the arc length method. Some numerical examples are also presented to ensure the validity of the program.

2.FORMULATION

The basic assumptions made are,

i).the normal to the shell mid surface remains straight and does not extend.
ii).the normal stresses in the direction of shell thickness are ignored.
iii).constitutive relation for an elasto-plastic material with small strains is considered.

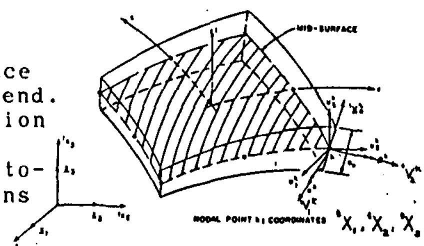

fig.(1)

Notation: Left subscript;configuration w.r.t.which the quantity is mesured.
Left superscript;configuration in which the quantity occurs.
No superscript;increment of the quantity

The principal of virtual displacement was used with Updated Lagrangian formulation.

Isoparametric formulation:The well known degenerated shell element was used<1>.9 node element used is shown in fig.(1).

The interpolation function used for a general shell is,

$$^tX_i=\sum_{k=1}^{9}(h_k)(^tX_i^k)+0.5\,t\sum_{k=1}^{9}(h_k)(a)(^tV_{ni}) \qquad (2.1)$$

Where , ^tX_i :Global coordinate of node k.
h_k :Shape function on r-s plane associated with node k.
a :Thickness of the shell.
$^tV_{ni}$:Component of the unit normal vector, $^t\mathbf{V}_n$, to the mid surface of the element at node k.

Initial values of V_n was computed as follows,

$$^0\mathbf{v}_n^k=\left(\frac{\partial \mathbf{x}}{\partial r}\times\frac{\partial \mathbf{x}}{\partial s}\right)\Big/\left|\frac{\partial \mathbf{x}}{\partial r}\times\frac{\partial \mathbf{x}}{\partial s}\right| \qquad (2.2)$$

The displacement increment is given by

$$_tu_i=\sum_{k=1}^{9}(h_k)(_tu_i^k)+0.5\,t\sum_{k=1}^{9}(h_k)(a)(^{t+\delta t}_{\ t}V_{ni}^k-{}^tV_{ni}^k) \qquad (2.3)$$

If two orthonormal vectors to Vni were selected as follows,

$$^t\mathbf{v}_1^k=(\mathbf{x}_2\times{}^t\mathbf{v}_n^k)/|\mathbf{x}_2\times{}^t\mathbf{v}_n^k| \ ; \quad {}^t\mathbf{v}_2^k={}^t\mathbf{v}_n^k\times{}^t\mathbf{v}_1^k \qquad (2.4)$$

and if α_k,β_k are the rotations of $^t\mathbf{v}_n$ about $^t\mathbf{v}_1$ and $^t\mathbf{v}_2$<2>,

$$^{t+\delta t}V_{ni}^k=0.5(\mathrm{Sin}\beta_k\mathrm{Cos}\alpha_k+\mathrm{Sin}\beta_k)\,{}^tV_{1i}^k-0.5(\mathrm{Sin}\alpha_k\mathrm{Cos}\beta_k+\mathrm{Sin}\alpha_k)\,{}^tV_{2i}^k$$
$$+\mathrm{Cos}\beta_k\mathrm{Cos}\alpha_k\,{}^tV_{ni}^k \qquad (2.5)$$

for small α and β , $${}^{t+\delta t}V^{k}_{ni}=\beta_{k}\,{}^{t}V^{k}_{1i}-\alpha_{k}\,{}^{t}V^{k}_{2i}+{}^{t}V^{k}_{ni} \qquad (2.6)$$

In FEM formulation, the basic unknowns are U_1, U_2, U_3, α and β .For further derivations refer to <1>.

Numerical Intigration:It is observed that the reduced intigration on shell surface gives accurate and effective solutions over higher order intigration.2X2 Gauss Legendre integration was employed.The Simpson's rule was used along the thickness direction.Maximum of 8 intervals was used between top and bottom.

Constitutive relation : i).Elasto-plastic behavior is modeled using Von Mises yield criteria and isotropic hardening.The flow theory with associated flow rule has been used.

ii).The normal stress along the thickness direction was made to zero by modifing the stress-strain relationship at local coordinate system.Due to this fact the costitutive relation is no longer isotropic.Therefore it is necessary to transform it to the global coordinate system before the stress calculations.

Solution algorithm:The popular research tool FEAP(Finite Element Analysis Program)was used to handle the basic finite element procedures.FEAP enables one to concentrate only on the particular structural element.In this study a fully general 3-D shell element subroutine was developed.The Modified Newton Raphson method for displacement increments and modified arc length method were examined.

3.NUMERICAL RESULTS

i)Non linear elastic;elasto-plastic

A plate with inplane loading was solved for linear elastic material and for elasto-plastic material.Results are compared with the previous results.fig.(2) shows the details.

ii)Elasto-plastic clamped beam

5 elements(9 node) were used in half span and was in good aggreement with the results of Argyris et al.<3> which was obtained from 50 frame elements in half span.Results are given in fig.(3).

Other numerical results will be presented on the day of conference.

4.CONCLUSION

This element shows very good accuracy even in extremely nonlinear problems.The problem of computational time must be overwhelmed by further studies.

REFERENCE:<1>Bathe K.J.,Finite element procedures in Eng.analysis ,pp 371-379 <2>Surana K.S.,Geometrically nonlinear formulation for the curved shell elements;Int.Journal for Num.Met.in Eng.,pp 581-615(1983) <3>Argyris J.H. et al,Natural analysis of elasto-plastic frames;Computer met.in Applied Mech.and Eng.,Vol.35,(1982).

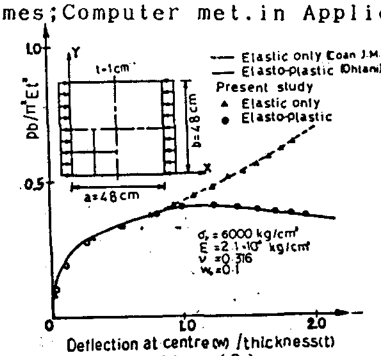

fig.(2).

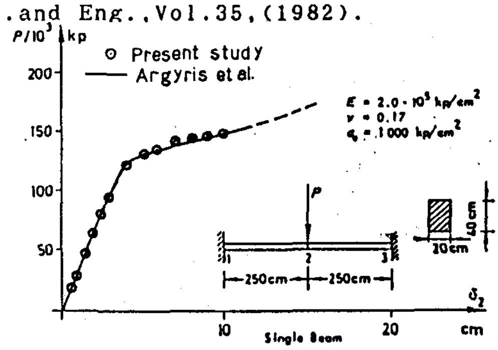

fig.(3)