B−2　長き水平部を有する傾斜管圧力計の液面の動きについて

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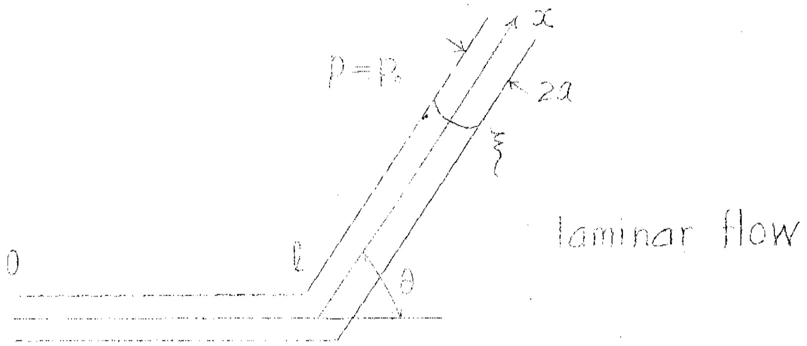


$p = f(t)$　　　$\ell/2a,\ \xi - \ell/2a \gg 1.$

$x,\ r,\ \vartheta$　　cylindrical coordinate

$u\ \ v_r\ \ v_\vartheta$

$v_r = v_\vartheta = 0,\quad \frac{\partial}{\partial\vartheta} = 0$

$\frac{\partial p}{\partial r} = 0$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\Delta u \qquad 0 < x < \ell$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -\frac{1}{\rho}\frac{\partial p}{\partial x} - g\,\mathrm{sine}\,\theta + \nu\Delta u \qquad \ell < x < \xi$$

$\frac{\partial u}{\partial x} = 0$

$\Delta = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}$

$\therefore\ u = u(x, t) \qquad p = p(r, t)$

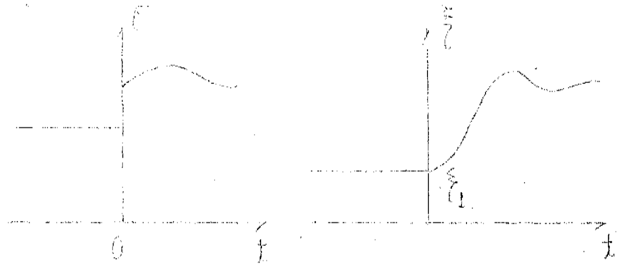
$$\begin{cases} x = 0, & p = f(t) \\ x = \xi, & p = p_0 \\ x = \ell, & p(\ell_{-0}, t) = p(\ell_{+0}, t) \end{cases}$$

$\frac{\partial u}{\partial t} - \nu\Delta u = \frac{d\phi}{dt}$

$$\phi(t) = \int_0^t \frac{f(t)}{\xi} - g\,\mathrm{sine}\,\theta\, dt$$

$$\sigma \cdots = \frac{p(t)}{\xi} - \frac{1}{\cdots} - g\,\sin\theta$$

Transient phenomena



$$u - \varphi = v$$
$$\frac{\partial v}{\partial t} - \nu \Delta v = 0$$
$$\begin{pmatrix} r = a & u = 0 \\ t \leq 0 & u = 0 \end{pmatrix}$$

$$\therefore \quad \left.\begin{array}{ll} v = -\varphi(t) & r = a \\ v = 0 & t \leq 0 \end{array}\right|$$

$$\therefore \quad v = \frac{2\nu}{a^2} \sum_{s=1}^{\infty} \lambda_s \frac{J_0\left(\lambda_s \frac{r}{a}\right)}{J_1(\lambda_s)} \int_0^t \varphi(\mu)\, e^{-\frac{\nu\lambda_s^2}{a^2}(t-\mu)}\, d\mu$$

$$\lambda_s > 0, \quad J_0(x) = 0$$

$$\frac{d\xi}{dt} = \frac{2}{a^2}\int_0^a u r\, dr$$

$$\left.\begin{array}{l} \dfrac{d\xi}{dt} = \dfrac{4\nu}{a^2} \sum_{s=1}^{\infty} \int_0^t \varphi(\mu)\, e^{-\frac{\nu\lambda_s^2}{a^2}(t-\mu)}\, d\mu + \varphi(t) \\ \dfrac{d\varphi}{dt} = \dfrac{\sigma(t)}{\xi} - g\,\sin\theta \qquad t \geq 0 \end{array}\right\}$$

$$t = 0, \quad \xi = \xi_1, \quad \varphi = 0$$

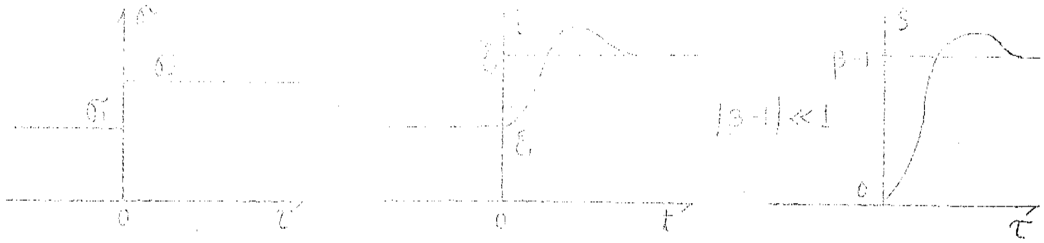
$$\left\{\begin{array}{l} \dfrac{d\xi}{dt} = 4\displaystyle\int \frac{\sigma(\mu)}{\xi(\mu)} \sum_{n=1}^{\infty} \frac{1}{\lambda_n^2}\, e^{-\frac{\lambda_n^2}{K^2}(t-\mu)}\, d\mu \\ \qquad - 4K^2 g \sin\theta \left\{ \dfrac{1}{32} - \displaystyle\sum_{n=1}^{\infty} \frac{1}{\lambda_n^4}\, e^{-\frac{\lambda_n^2}{K^2} t} \right\} \\ t = 0, \quad \xi = \xi_1 \qquad t \geq 0 \end{array}\right.$$

$$K = \sqrt{\frac{a^2}{\nu}} \qquad \frac{t}{K^2} = \tau \qquad L\{F\} = \int_0^{\infty} e^{-z\tau} F(\tau)\, d\tau$$

$$\zeta = \frac{\xi}{\xi_1} \qquad \beta = \frac{\xi_2}{\xi_1} \qquad \alpha = \xi_1^2 / K^4 \sigma_2 = \frac{\xi_1}{\xi_2}\,\frac{\xi_1}{K^4 g \sin\theta}$$

$$L\{\zeta\} = \cdots \frac{J_2(i\sqrt{z})}{J_0(i\sqrt{z})} \qquad L\{1/1+\zeta\} = \frac{1}{\beta}\,\frac{1}{z^2}\,\frac{J_2(i\sqrt{z})}{J_0(i\sqrt{z})}$$

$$\zeta = \sum_{n=1}^{\infty} \left(\frac{\beta-1}{\beta}\right)^n \zeta_r(\tau, \beta)$$



$$\therefore \alpha L\left\{ \frac{\beta-1}{\beta}\zeta_1 + \left(\frac{\beta-1}{\beta}\right)^2 \zeta_2 + \cdots \right\} =$$

$$-\frac{1}{Z^2}\frac{J_2(i\sqrt{Z})}{J_0(i\sqrt{Z})} L\left\{1 - \frac{\beta-1}{\beta}\zeta_1 + \left(\frac{\beta-1}{\beta}\right)^2(\zeta_1^2 - \zeta_2) + \cdots\right\}$$

$$+\frac{1}{\beta}\frac{1}{Z^3}\frac{J_2(i\sqrt{Z})}{J_0(i\sqrt{Z})}$$

$$\therefore L\{\zeta_1\} = \frac{1}{Z} J_2(i\sqrt{Z}) \Big/ J_2(i\sqrt{Z}) - \alpha Z^2 J_0(i\sqrt{Z})$$

$$L\{\zeta_2\} = L\{\zeta_1^2\} J_2(i\sqrt{Z}) \big/ J_2(i\sqrt{Z}) - \alpha Z^2 J_0(i\sqrt{Z})$$

$L\{\zeta_1\}$ pole of 1st order

$Z = 0, -b_1^2, -b_2^2, -C_1^2, -C_2^2, \cdots\cdots, -C_n^2, \cdots$

$0 < C_1 < C_2 < \cdots < C_n < \cdots$

$\lambda_{0n} < C_n < \lambda_{2n} \qquad n = 1, 2, \cdots\cdots$

$\lambda_{0n} : J_0(x) = 0 \qquad \lambda_{2n} : J_2(x) = 0$

$\lambda_{0n} \doteqdot \lambda_{2n} \qquad C_1 \cong 5 \qquad C_2 \cong 8.5$

$\alpha > \alpha_0 = 0.084\ 728 \cdots$

$0 < b_1 < b_2 < \lambda_{01} \qquad \lambda_{01}$ 1st Positive root

$\alpha = \alpha_0$

$0 < b_1 = b_2 < \lambda_{01} \quad (b_1 = b_2 = 1.709\ 7 \cdots)$

$\alpha < \alpha_0$

$\overline{b}_1 = b_2 \qquad \alpha_0/\alpha = E$

$0 < \Re\{b_1\} < \lambda_{01}$

$1 \geqq \alpha_0/\alpha$ aperiodic

$1 < \alpha_0/\alpha$ oscillatory

$$\zeta_1 = 1 + 4\left\{\frac{\alpha b_1^2}{(1+\alpha b_1^4)^2 - 12\alpha b_1^2} e^{-b_1^2\tau} + \frac{\alpha b_2^2}{(1+\alpha b_2^4)^2 - 12\alpha b_2^2} e^{-b_2^2\tau}\right\}$$

$$+4\sum_{n=1}^{\infty} \frac{\alpha C_n^2}{(1+\alpha C_n^4)^2 - 12\alpha C_n^2} e^{-C_n^2\tau}$$

$\tau = 0,\ \xi_1 = 0,\quad d\xi_1/d\tau = 0$

$\tau = +\infty\ \{(\beta-1)/\beta\}\xi_1 = (\beta-1)/\beta\quad$ (c.f. $\xi = \beta - 1$)

$\nu = 10^{-2}\ cm^2/s\quad \zeta_1 = 5\times10^2\ cm\quad \zeta_2 = 5.5\times10^2\ cm$

$g = 980\ cm/s^2\quad \sin\theta = 1/10\quad a = 2\times10^{-1}\ cm$

$\alpha_0/\alpha = E = 0.291 < 1\quad$ aperiodic

Poiseuille approximation

$$\frac{d^2\zeta}{dt^2} + \frac{8}{K^2}\frac{d\zeta}{dt} + g\sin\theta = \sigma(t)/\zeta\qquad t \geqq 0$$

$t = 0\quad \zeta = \zeta_1\quad d\zeta/dt = 0$

$\sigma(t) = \sigma_2\qquad \zeta \sim \zeta_1,\ \zeta_2$

$\zeta = \zeta_2 - (\zeta_2 - \zeta),\ \frac{1}{\zeta} \doteqdot \frac{1}{\zeta_2} + \frac{1}{\zeta_2^2}(\zeta_2 - \zeta)$

$$\frac{d^2}{dt^2}(\zeta - \zeta_2) + \frac{8}{K^2}\frac{d}{dt}(\zeta - \zeta_2) + \frac{\sigma_2}{\zeta_2^2}(\zeta - \zeta_2) = 0$$

$m_1 = \frac{4}{K^2}(1 + \sqrt{1-F}),\quad m_2 = \frac{4}{K^2}(1 - \sqrt{1-F}),$

$F = \frac{1}{16}\frac{\sigma_2 K^4}{\zeta_2^2}\ \left(= 0.0625\frac{K^4\sigma_2}{\zeta_2^2}\right)\qquad$ c.f. $E = 0.0847\frac{K^4\sigma_2}{\zeta_1^2}$

$F \leqq 1\quad$ aperiodic $\quad F > 1\quad$ oscillatory

$\zeta = \zeta_2\{1 + x_1(t)\}$

$$x_1(t) = \frac{\zeta_2 - \zeta_1}{\zeta_2}\frac{2}{K^2}\frac{F}{\sqrt{1-F}}\left(\frac{e^{-m_1 t}}{m_1} - \frac{e^{-m_2 t}}{m_2}\right)$$

$(t = +\infty,\ \zeta = \zeta_2)$

$$\frac{d^2\zeta}{dt^2} + \frac{8}{K^2}\frac{d\zeta}{dt} + g\sin\theta = \frac{\sigma_2}{\zeta_2\{1 + x_1(t)\}} - \frac{\sigma_2}{\zeta_2}\{1 - x_1(t)\} + \frac{\sigma_2}{\zeta_2}\left(1 + \frac{\zeta_2 - \zeta}{\zeta_2}\right)$$

$$\frac{d^2}{dt^2}(\zeta - \zeta_2) + \frac{8}{K^2}\frac{d}{dt}(\zeta - \zeta_2) + \frac{\sigma_2}{\zeta_2^2}(\zeta - \zeta_2) = \frac{\sigma_2}{\zeta_2}\frac{x_1^2(t)}{1 + x_1(t)}$$

$t = 0\quad \zeta = \zeta_1\quad d\zeta/dt = 0$

$\zeta = \zeta_2\{1 + x_1(t) + x_2(t)\}$

$$x_2 = -\frac{2F}{K^2\sqrt{1-F}}\int_0^t\left\{e^{-m_1(t-\mu)} - e^{-m_2(t-\mu)}\right\}\frac{x_1^2(\mu)}{1 + x_1(\mu)}\,d\mu$$

$t = +\infty,\quad \zeta = \zeta_2$