Numerical Simulation of Oil Displacement by water in Petroleum Reservoir

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1. Introduction

In petroleum reservoir engineering, the technique of injecting water into a reservoir is used to maintain the oil production from reservoirs. This is known as the waterflooding technique, and it provides high oil production rates and a high degree of petroleum recovery. When water is injected into a reservoir, oil is displaced toward production well in a situation of two-phase flow[1].

The analysis of immiscible displacements of two-phase fluids has been studied extensively in the discipline of fluid flow through porous media, and an ingenious and simple approach to this problem was provided by Buckley and Leverett[2]. In this paper, Buckley-Leverett analysis is performed first, and the finite difference analysis is attempted using the same physical data adopted in the Buckley-Leverett analysis.

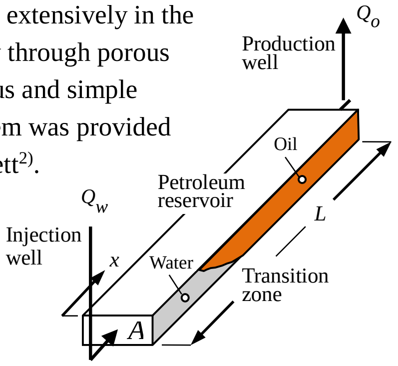

Fig.1 Waterflooding method for oil production[1].

2. Buckley-Leverett Analysis

In the first, the Buckley-Liverett frontal displacement theory is reviewed, and an example of the analysis is presented. The flow rate of oil and water, through completely saturated porous media in horizontal direction is given by the Darcy's law as follows

$$Q_o=-\frac{k_x k_{ro}A}{\mu_o}\frac{\partial p_o}{\partial x}\quad,\quad Q_w=-\frac{k_x k_{rw}A}{\mu_w}\frac{\partial p_w}{\partial x}\qquad(1)(2)$$

Where k_x is the intrinsic permeability of the medium, A is the cross-sectional area for permeation, μ_o and μ_w are the dynamic viscosity of oil and water, p_o and p_w are the pore pressure of oil and water, and k_{ro} and k_{rw} are the relative permeability of oil and water respectively. The relative permeability k_{ro} and k_{rw} are given as a function of water saturation S_w as shown in Fig.2. Subtracting Eq.(1) from Eq.(2) and substituting the relation of $Q_o=Q_T-Q_w$, it becomes

$$Q_T=Q_w\left(1+\frac{k_{ro}}{k_{rw}}\frac{\mu_w}{\mu_o}\right)-\frac{k_x k_{ro}A}{\mu_o}\frac{\partial p_{o/w}}{\partial x}\qquad(3)$$

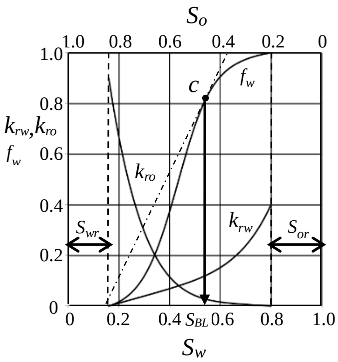

Fig.2 Relative permeability curves.

where $p_{o/w}=p_w-p_o$ is a capillary pressure between oil and water. Owing to very small capillary pressure, $p_{o/w}$ is neglected, then the fraction of pore water flow, i.e., fractional flow rate $f_w=Q_w/Q_T$ is expressed as

$$f_w=\frac{1}{1+\frac{k_{ro}}{k_{rw}}\frac{\mu_w}{\mu_o}}\qquad(4)$$

Next, continuity equation of pore water is expressed as

$$-\frac{\partial Q_w}{\partial x}=A\phi\frac{\partial S_w}{\partial t}\qquad(5)$$

where ϕ is the porosity of the reservoir. Using the relations of $Q_w=f_wQ_T$ and $f_w(S_w)$, Eq.(5) is rewritten as

$$\left(\frac{\partial S_w}{\partial t}\right)_x=-\frac{Q_T}{A\phi}\left(\frac{df_w}{dS_w}\frac{\partial S_w}{\partial x}\right)_t\qquad(6)$$

The main and very productive idea of Buckley-Leverett is to transform Eq.(6) into the following form:

$$\left(\frac{\partial x}{\partial t}\right)_{S_w}=\frac{Q_T}{A\phi}\left(\frac{df_w}{dS_w}\right)_t\qquad(7)$$

stating that the rate of advance of a plane of fixed saturation S_w is proportional to the rate of change in composition of the flowing stream with saturation. Eq.(7) can be integrated to give the position of a particular saturation as a function of time.

$$x_{S_w}=\frac{Q_Tt}{A\phi}\left(\frac{df_w}{dS_w}\right)\qquad(8)$$

Table1 Relative permeabilities and fractional flow function for Buckley-Leverett problem.(after Aziz and Settari[3])

S_w	k_{rw}	k_{ro}	$f_w=[1+\frac{k_{ro}}{k_{rw}}]^{-1}$	$f'_w=df_w/dS_w$
0.00	0.000	0.900	0.000	0.000
0.16	0.000	0.900	0.000	0.425
0.20	0.012	0.675	0.017	0.680
0.25	0.025	0.465	0.051	1.260
0.30	0.040	0.310	0.114	1.880
0.35	0.055	0.210	0.208	3.020
0.40	0.070	0.125	0.359	3.780
0.45	0.085	0.070	0.548	4.040
0.50	0.105	0.035	0.750	1.740
0.55	0.128	0.025	0.837	1.020
0.60	0.158	0.020	0.888	0.780
0.65	0.190	0.015	0.927	0.660
0.70	0.240	0.010	0.960	0.480
0.75	0.310	0.005	0.984	0.320
0.80	0.410	0.000	1.000	0.000
1.00	0.410	0.000	1.000	

Residual water saturation $S_{wr}=0.16$
Residual oil saturation $S_{or}=0.20$

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As a quantitative demonstration for the Backley-Leverett analysis, relative permeability data shown in Table1 was used. For a situation of the reservoir porosity ϕ=0.2, cross-sectional area for permeation A=100m^2, reservoir length L=100m and the amount of water injected into reservoir Q_T=Q_w=1m^3/day, the calculated results of saturation profile is shown in Fig.3.

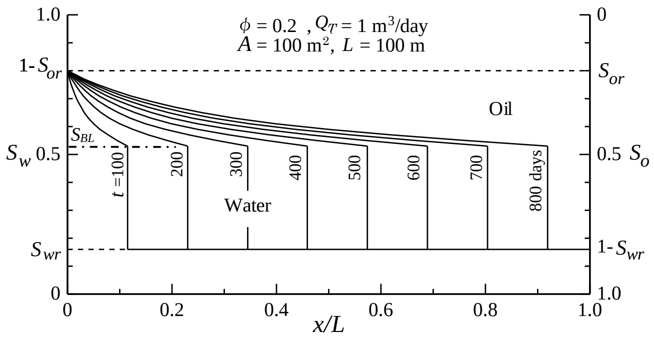

Fig.3 Calculated results of saturation profile by Buckley-Leverett analysis.

3. Numerical Simulation

In the next, the Buckley-Leverett oil displacement problem is solved via numerical methods. The equations of flow for water and oil in an isotropic-homogenous incompressible medium are written as

$$\frac{\partial}{\partial x}\left(k_x \lambda_w \frac{\partial p_w}{\partial x}\right) = \phi \frac{\partial S_w}{\partial t} \qquad (9)$$

$$\frac{\partial}{\partial x}\left(k_x \lambda_o \frac{\partial p_o}{\partial x}\right) = \phi \frac{\partial S_o}{\partial t} \qquad (10)$$

where $\lambda_w = k_{rw}/\mu_w$ and $\lambda_o = k_{ro}/\mu_o$ are mobility factors, and p_w and p_o are fluid pressures in each phase. A common axial relationship available between saturation and pressure;

$$S_w + S_o = 1 \quad , \quad p_o - p_w = p_{o/w} \qquad (11a,b)$$

If the capillary pressure $p_{o/w}$ is neglected, i.e. $p_o = p_w$, the water and oil pressures are replaced by a single variable p. By adding the water and oil phase equations of Eq.(9) and Eq.(10), the equation of two-phase flow is described as

$$\frac{\partial}{\partial x}\left(k_x \lambda_w \frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial x}\left(k_x \lambda_o \frac{\partial p}{\partial x}\right) = 0 \qquad (12)$$

Hereupon, time derivative term of saturation has been eliminated, Eq.(12) is called the pressure equation. The boundary conditions may be given by

$$-k_x \lambda_w \frac{\partial p}{\partial x} = \frac{\hat{Q}_T}{A} \quad \text{at} \quad x = 0 \quad , \quad p = \hat{p} \quad \text{at} \quad x = L \qquad (13a,b)$$

The pressure equation (12) is transformed to the finite difference equation, and the resulting algebraic equation can

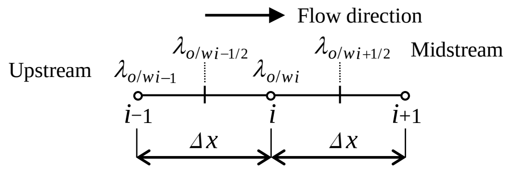

Fig.4 Evaluation points of the mobility factor.

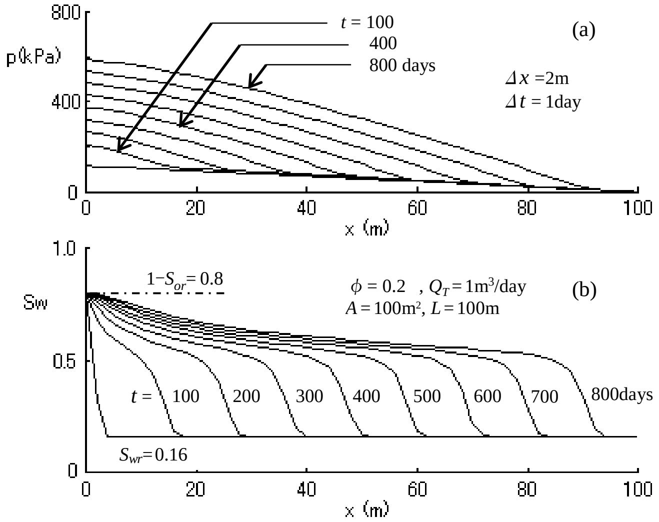

Fig.5 Calculated results of the Buckley-Leverett problem by FDM: (a) Pressure, (b) Saturation profile.

be written in the general form as

$$-A_i p_{i+1}^{k+1/2} + B_i p_i^{k+1/2} - C_i p_{i-1}^{k+1/2} = D_i \qquad (14)$$

where

$$A_i = \gamma(\lambda_{wi} + \lambda_{oi}) \quad , \quad B_i = A_i + C_i$$
$$C_i = \gamma(\lambda_{wi-1} + \lambda_{oi-1}) \quad , \quad D_i = 0 \qquad (15a,b,c,d)$$

in which $\gamma = k_x \Delta t / \Delta x^2$. The mobility factors $\lambda_{o/w}$ are normally evaluated at the upstream side nodes as shown in Fig.4, i.e. upstream weighting, to obtain stable solution. Equation (14) can be solved implicitly associated with the boundary conditions of (13a,b) using the tridiagonal matrix solver. Once pressure is computed, water saturation can be calculated explicitly as follows.

$$S_{wi}^{k+1} = \frac{\gamma}{\phi}\left\{\lambda_{wi}\, p_{i+1}^{k+1/2} - (\lambda_{wi} + \lambda_{wi-1})\, p_i^{k+1/2} + \lambda_{wi-1} p_{i-1}^{k+1/2}\right\} + S_{wi}^{k} \qquad (16)$$

The pressure and saturation equation because of high nonlinearity, computations of Eqs.(14)(16) must be iterated until successive changes in $p_i^{k+1/2}$ and S_{wi}^{k+1} are settled in the prescribed tolerances.

The FDM calculation result, based on the same physical data of the Buckley-leverett, is illustrated in Fig.5. A good coincidence is seen between numerical analysis and Buckley-leverett analysis. Moreover, the pressure profile can be obtained in FDM that provide sufficient information for the waterflooding operation in petroleum reservoir engineering technology.

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