Optimization Technique for Groundwater Management

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1. Introduction

Management of groundwater resource systems is necessary in many countries to assure a sustainable water supply at the national, regional, and community levels. In Afghanistan, the author's homeland, the static water level is decreasing yearly, especially in expanding population areas, where groundwater usage is extremely high. For example, the groundwater level in Kabul City has decreased by about 10 meters over the last decade, mainly due to unplanned pumping by private users concentrated in that area. A linear programming (LP) technique based on the simplex method is applied to the groundwater management problem to obtain maximum benefits, forming an optimal management policy under hydrological constraints.

2. Groundwater Drawdown in Multiple Well Systems

The partial differential equation for a steady flow to the well in a radial coordinate system takes the form[1)]

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r} = 0 \tag{1}$$

where h is the hydraulic head around the well. The boundary conditions at the well are $r = r_w$, $h = h_w$, and at some distance $r = R$, $h = H$. The distance R, where the drawdown is zero or negligible, is called the radius of the influence circle. Integrating Eq.(1) from r to R with the boundary conditions,

$$s(r) = H(R) - h(r) = \frac{Q_w}{2\pi T}\ln\left(\frac{R}{r}\right) \tag{2}$$

where $s(r)$ is the drawdown at distance r, Q_w is the pumping rate and $T = KB$ is the transmissivity of the aquifer.

When wells are spaced at distances smaller than their radius of influence R, they affect each other's drawdown as shown in Fig.1.

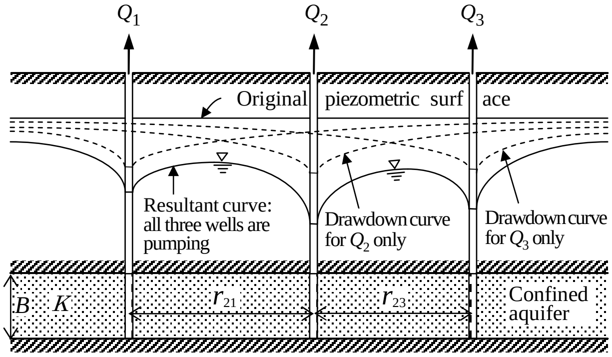

Fig.1 Composite drawdown curves by multiple wells.

Because the equation for flow in a confined aquifer is linear in $h(r)$, the principle of superposition is applicable. In a confined aquifer in which N wells are operating at constant pumping rates, letting Q_i denote the pumping rate at point $(x_{w,i}, y_{w,i})$, and R_i the influence circle of wells, the total drawdown at location (x_j, y_j) is

$$s_j = \sum_{i=1}^{N}\left\{\left(\frac{Q_i}{2\pi T}\right)\ln\left(\frac{R_i}{\sqrt{(x_j - x_{w,i})^2 + (y_j - y_{w,i})^2}}\right)\right\} \tag{3}$$

where the distance r between a pumping site and observation point is $r = \sqrt{(x_j - x_{w,i})^2 + (y_j - y_{w,i})^2}$.

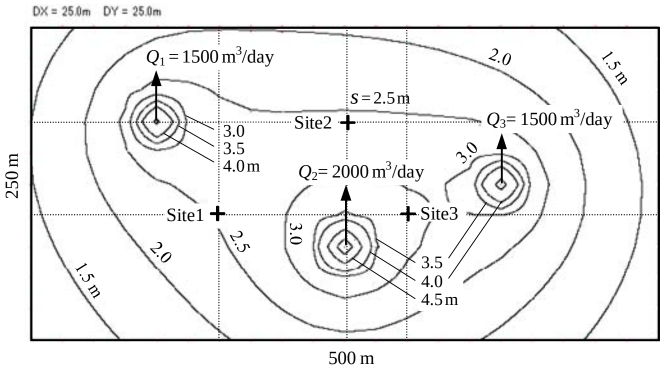

Fig.2 Cones of depression for three pumping wells.

Fig.2 illustrates the cones of depressions by three wells, each pumping at constant rates of $Q_1 = 1500$, $Q_2 = 2000$ and $Q_3 = 1500$ m^3/day from a subsurface confined aquifer of thickness $B = 5$m and hydraulic conductivity $K = 1\times10^{-3}$m/s (transmissivity $T = 5\times10^{-3}$ m^2/s). The contour lines in Fig.2 indicate the drawdowns mutually influenced due to simultaneous pumping by three wells.

3. Application of the Linear Programing Technique to Groundwater Management

Generally, the optimization problem is characterized by an objective function, stating the quantity to be maximized or minimized and its functional dependence on decision variables, and by constraints on the decision variables among which an optimum is to be found. In the groundwater management problem, the objective function and constraints of the pumping rates Q_i of N wells are the decision variables, and the total amount of pumping is the objective function to be maximized. The constraints demand that at M locations (x_j, y_j) the drawdowns s_j must be smaller than some given maximum drawdown $s_{j,\max}$ to

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avoid problems such as drying up neighboring wells or excessive land subsidence in the area due to pumping. The problem is now described as:

$$\text{Objective function:}\quad Z=\sum_{i=1}^{N} Q_i \;\rightarrow\; \text{Maximize}$$

$$\text{Constraints:}\quad s_j(Q_1\cdots Q_N)\le s_{j,\max}\;(j=1,\cdots,M)$$

$$Q_i\ge 0\quad (i=1,\cdots\cdots,N) \tag{4}$$

The functional relationship between drawdowns and pumping rates is given by the analytical formula Eq.(3). The constraints for drawdown in Eq.(4) are rewritten as

$$\sum_{i=1}^{N} Q_i a_{ij}\le s_{j,\max}\quad (j=1,\cdots,M) \tag{5}$$

$$\text{with}\quad a_{ij}=\left(\frac{1}{2\pi T}\right)\ln\left(\frac{R_i}{\sqrt{(x_j-x_{w,i})^2+(y_j-y_{w,i})^2}}\right) \tag{6}$$

The matrix a_{ij} is called the influence matrix and gives the change in drawdown at location j if the pumping rate at well i is increased by one unit. The optimization problem is linear as long as the objective function and constraints are linear (i.e., linearity of the system). Therefore, the optimization model of Eq.(4) takes the form of a standard linear optimization problem that can be solved by the simplex algorithm[2]. The standard form of linear optimization is given by the expressions

$$\text{Maximize}\quad Z=\sum_{i=1}^{n} p_i x_i \tag{7}$$

$$\text{Subject to}\quad \sum_{j=1}^{n} a_{ij}x_j\le b_i\quad (i=1,\cdots,m)$$

$$b_i\ge 0$$

$$x_j\ge 0\quad (j=1,\cdots,n)$$

in which Z is the objective function, x_i are the decision variables ($i=1,\cdots,n$), and p_i are the benefit coefficients. The system of inequalities can be changed into a system of equations by introducing non-negative slack variables, and optimum solution can be found as a feasible solution by exchanging basis variables and non-basis variables until all the coefficients of the objective function become no negative values[3]. As an application of LP to the groundwater management problem, the multiple well pumping system shown in Fig.2 is considered. The locations of the three pumping wells are, taking the top-left corner of the region as the coordinate origin, $(x_{w,1}, y_{w,1})$ = (100 m,75 m) , $(x_{w,2}, y_{w,2})$ = (250 m,175 m) and $(x_{w,3}, y_{w,3})$ = (375 m,125 m). At three sites, Site1 (150 m ,150 m), Site2 (250 m ,75 m) and Site3 (300 m ,150 m), maximum allowable drawdowns are prescribed as $s_1\le 2.5$ m, $s_2\le 2.5$ m and $s_3\le 3.0$ m. The radii of influence are assumed to be R_i = 500 m for every well. The objective is now to pump as much water as possible without violating the constraints.

If these data are substituted into Eqs.(5) and (6), the optimization problem is expressed by the tableau of a canonical form shown in the next. (the unit of a_{ij} is s/m^2)

Initial Tableau

	a_1	a_2	a_3	a_4	a_5	a_6	b
	54.56	50.29	25.23	1	0	0	2.5
	38.34	51.26	41.79	0	1	0	2.5
	27.09	69.78	58.74	0	0	1	3.0
Z	−1	−1	−1	0	0	0	0

m=3
n=6
: Pivot element.

First Tableau

35.04	0	-17.10	1	0	-0.7207	0.3379
18.44	0	-1.360	0	1	-0.7346	0.2962
0.3882	1	0.8418	0	0	0.01433	0.04299
−0.6118	0	−0.1582	0	0	0.01433	0.04299

Second Tableau

1	0	-0.4882	0.02854	0	-0.02057	0.00965
0	0	7.642	-0.5263	1	-0.3553	0.1184
0	1	1.031	-0.1108	0	0.02232	0.03925
0	0	−0.4569	0.01746	0	0.00175	0.04889

Fifth Tableau

1	0.6158	0	0.03183	-0.0192	0	0.03154
0	0.6617	1	-0.0292	0.0416	0	0.03089
0	14.232	0	0.8532	-1.921	1	0.3312
0	0.2774	0	0.00263	0.0223	0	0.06243

Since the last row has no negative elements, we conclude that the solution corresponding to the fifth tableau is optimal. Therefore, the optimal pumping rates are Q_1= 0.03154 m^3/s = 2720 m^3/day, Q_2 = 0, Q_3= 0.03089 m^3/s = 2670 m^3/day and total amount of feasible pumping is 5390 m^3/day. The best policy is to stop pumping Q_2 and increase Q_1 and Q_3 up to the optimal values. The cones of depressions for optimum pumping operation are depicted in Fig.3.

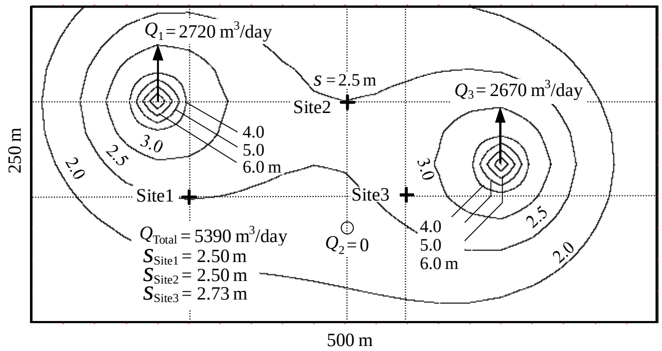

Fig.3 Cones of depression for optimum pumping rates.

4. Summary

An LP technique was applied to the groundwater management problem to find optimum pumping operations. The technique can be employed in groundwater resource system planning and analysis in many developing countries.

References: 1) Bear J.: Hydraulics of groundwater, (Mc-Graw Hill, 1979) p.304-306. 2) Kinzelbach W.: Groundwater Modeling, Development in Water Science 25 (Elsevier, 1986) p.188-220. 3) Luenberger D.G.: Introduction to Linear and Nonlinear Programming (Addison-Wesley, 1973), p.27-63.