Analysis of Waterflooding in Petroleum Reservoir

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1. Introduction

In petroleum reservoir engineering, a technique of injecting water into reservoir is used in order to maintain oil production rates during the pumping operation. The method is known as the waterflooding technique, which provides high oil production rates and high degree of petroleum recovery when oil production rates deteriorate (Fig.1).

The Buckley-Leverett frontal displacement theory [2] is performed for calculating the saturation profile based on the relative permeability data employed by Azis and Settari[3]. Also, the numerical analysis based on the method of IMPES* is attempted using the same physical data adopted in the Buckley-Leverett analysis.

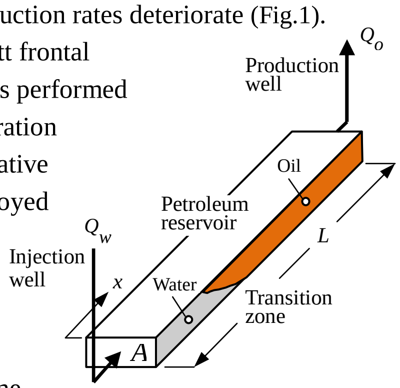

Fig.1 Waterflooding method for oil production[1].

(*Implicit Pressure and Explicit Saturation.)

2. Buckley-Leverett Analysis

In the first, the Buckley-Liverett frontal displacement theory is reviewed, and an example of the analysis is presented. The flow rate of oil and water through completely saturated porous medium in horizontal direction is given by the Darcy's law as follows.

$$Q_o = -\frac{k_x k_{ro} A}{\mu_o}\frac{\partial p_o}{\partial x} \quad , \quad Q_w = -\frac{k_x k_{rw} A}{\mu_w}\frac{\partial p_w}{\partial x} \qquad (1)(2)$$

where k_x is the intrinsic permeability of the medium, A is the cross-sectional area for permeation, μ_o and μ_w are the dynamic viscosity of oil and water, p_o and p_w are the pore pressure of oil and water, and k_{ro} and k_{rw} are the relative permeability of oil and water respectively. The relative permeability k_{ro} and k_{rw} are given as a function of water saturation S_w as shown in Fig.2. Subtracting Eq.(1) from Eq.(2) and substituting the relation of $Q_o = Q_T - Q_w$, it becomes

$$Q_T = Q_w\left(1 + \frac{k_{ro}}{k_{rw}}\frac{\mu_w}{\mu_o}\right) - \frac{k_x k_{ro} A}{\mu_o}\frac{\partial p_{o/w}}{\partial x} \qquad (3)$$

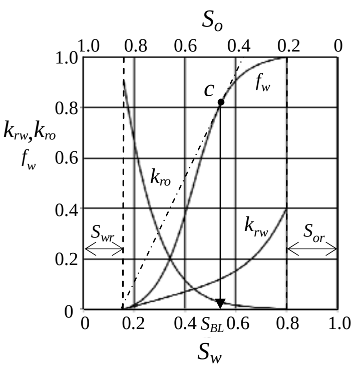

Fig.2 Relative permeability curves.

where $p_{o/w} = p_w - p_o$ is the capillary pressure between oil and water. If the effect of capillary pressure is neglected, the fraction of pore water flow, i.e., fractional flow rate f_w, is expressed as

$$f_w = \frac{1}{1 + \dfrac{k_{ro}}{k_{rw}}\dfrac{\mu_w}{\mu_o}} \qquad (4)$$

Next, continuity equation of pore water is expressed as

$$-\frac{\partial Q_w}{\partial x} = A\phi\frac{\partial S_w}{\partial t} \qquad (5)$$

where ϕ is the porosity of the reservoir. Using the relations of $Q_w = f_w Q_T$ and $f_w(S_w)$, Eq.(5) is rewritten as

$$\left(\frac{\partial S_w}{\partial t}\right)_x = -\frac{Q_T}{A\phi}\left(\frac{df_w}{dS_w}\frac{\partial S_w}{\partial x}\right)_t \qquad (6)$$

The main and very fruitful idea of Buckley and Leverett is to transform Eq.(6) into the following form:

$$\left(\frac{\partial x}{\partial t}\right)_{S_w} = \frac{Q_T}{A\phi}\left(\frac{df_w}{dS_w}\right)_t \qquad (7)$$

stating that the rate of advance of a plane of fixed saturation S_w is proportional to the rate of change in composition of the flowing stream with saturation. Eq.(7) can be integrated to give the position of a particular saturation as a function of time.

$$x_{S_w} = \frac{Q_T t}{A\phi}\left(\frac{df_w}{dS_w}\right) \qquad (8)$$

Table1 Relative permeabilities and fractional flow function for Buckley-Leverett problem.(after Aziz and Settari[3])

S_w	k_{rw}	k_{ro}	$f_w = [1+\frac{k_{ro}}{k_{rw}}]^{-1}$	$f'_w = df_w/dS_w$
0.00	0.000	0.900	0.000	0.000
0.16	0.000	0.900	0.000	0.425
0.20	0.012	0.675	0.017	0.680
0.25	0.025	0.465	0.051	1.260
0.30	0.040	0.310	0.114	1.880
0.35	0.055	0.210	0.208	3.020
0.40	0.070	0.125	0.359	3.780
0.45	0.085	0.070	0.548	4.040
0.50	0.105	0.035	0.750	1.740
0.55	0.128	0.025	0.837	1.020
0.60	0.158	0.020	0.888	0.780
0.65	0.190	0.015	0.927	0.660
0.70	0.240	0.010	0.960	0.480
0.75	0.310	0.005	0.984	0.320
0.80	0.410	0.000	1.000	0.000
1.00	0.410	0.000	1.000	

Residual water saturation $S_{wr} = 0.16$
Residual oil saturation $S_{or} = 0.20$

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As a quantitative demonstration for the Backley-Leverett analysis, relative permeability data shown in Table1 was used. For a situation of the reservoir porosity $\phi = 0.2$, cross-sectional area for permeation $A = 100\,\mathrm{m}^2$, reservoir length $L = 100\,\mathrm{m}$ and the amount of water injected into reservoir $Q_T = Q_w = 1\mathrm{m}^3$/day, the calculated results of saturation profile is shown in Fig.3.

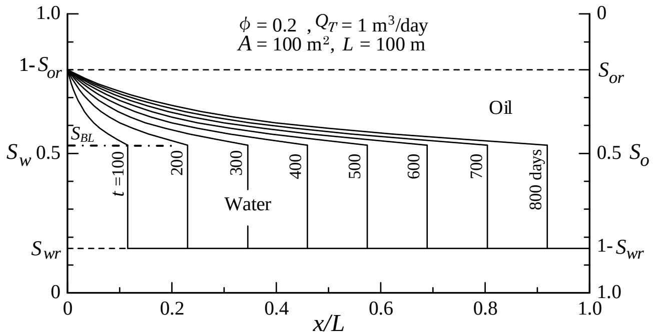

Fig.3 Calculated results of saturation profile by Buckley-Leverett analysis.

3. Numerical Analysis

In the next, the Buckley-Leverett oil displacement problem is solved via numerical methods. The equations of flow for water and oil in an isotropic-homogenous incompressible medium are written as

$$\frac{\partial}{\partial x}(k_x \lambda_w \frac{\partial p_w}{\partial x}) = \phi \frac{\partial S_w}{\partial t} \qquad (9)$$

$$\frac{\partial}{\partial x}(k_x \lambda_o \frac{\partial p_o}{\partial x}) = \phi \frac{\partial S_o}{\partial t} \qquad (10)$$

where $\lambda_w = k_{rw}/\mu_w$ and $\lambda_o = k_{ro}/\mu_o$ are mobility factors, and p_w and p_o are fluid pressures in each phase. There exists the following axially relationships for saturation and pressure;

$$S_w + S_o = 1 \quad , \quad p_o - p_w = p_{o/w} \qquad (11a,b)$$

If the capillary pressure $p_{o/w}$ is neglected, i.e. $p_o = p_w$, the water and oil pressures are replaced by a single variable p. By adding the water and oil phase equations of Eq.(9) and Eq.(10), the equation of two-phase flow is described as

$$\frac{\partial}{\partial x}(k_x \lambda_w \frac{\partial p}{\partial x}) + \frac{\partial}{\partial x}(k_x \lambda_o \frac{\partial p}{\partial x}) = 0 \qquad (12)$$

Hereupon, time derivative term of saturation has been eliminated, Eq.(12) is called the pressure equation. The boundary conditions may be given by

$$-k_x \lambda_w \frac{\partial p}{\partial x} = \frac{\hat{Q}_T}{A} \quad \text{at} \quad x = 0 \quad , \quad p = \hat{p} \quad \text{at} \quad x = L \qquad (13a,b)$$

The pressure equation (12) is transformed to the finite difference equation, and the resulting algebraic equation can

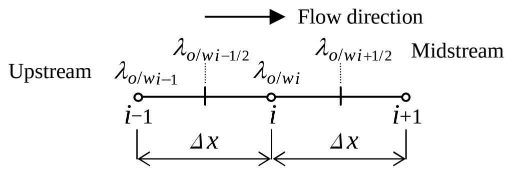

Fig.4 Evaluation points of the mobility factor.

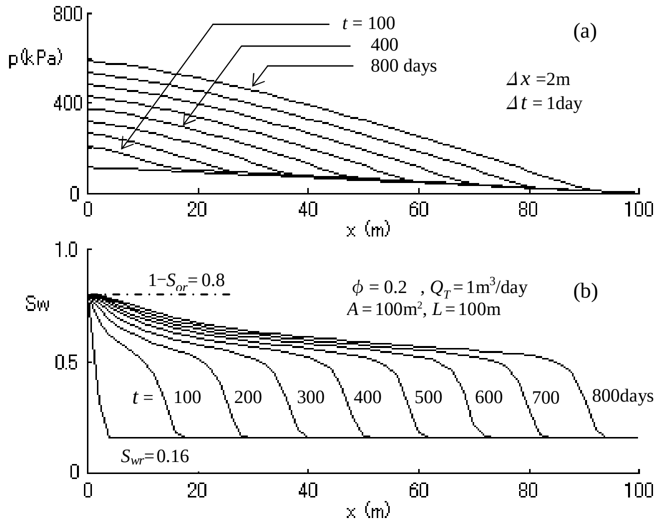

Fig.5 Calculated results of the Buckley-Leverett problem by FDM: (a) Pressure, (b) Saturation profile.

be written in the general form as

$$-A_i p_{i+1}^{k+1/2} + B_i p_i^{k+1/2} - C_i p_{i-1}^{k+1/2} = D_i \qquad (14)$$

where

$$A_i = \gamma(\lambda_{wi} + \lambda_{oi}) \quad , \quad B_i = A_i + C_i$$
$$C_i = \gamma(\lambda_{wi-1} + \lambda_{oi-1}) \quad , \quad D_i = 0 \qquad (15a,b,c,d)$$

in which $\gamma = k_x \Delta t / \Delta x^2$. The mobility factors $\lambda_{o/w}$ are normally evaluated at the upstream side nodes as shown in Fig.4, i.e. upstream weighting, to obtain stable solution. Equation (14) can be solved implicitly associated with the boundary conditions of (13*a*,*b*) using the tridiagonal matrix solver. Once pressure is computed, water saturation can be calculated explicitly as follows.

$$S_{wi}^{k+1} = \frac{\gamma}{\phi}\{\lambda_{wi}\, p_{i+1}^{k+1/2} - (\lambda_{wi} + \lambda_{wi-1})\, p_i^{k+1/2} + \lambda_{wi-1} p_{i-1}^{k+1/2}\} + S_{wi}^{k} \qquad (16)$$

Owing to high nonlinearity, computations of Eqs.(14)(16) must be iterated until successive changes in $p_i^{k+1/2}$ and S_{wi}^{k+1} are settled in the prescribed tolerances.

Calculated result by FDM under the same reservoir condition with the Buckley-Leverett analysis is shown in Fig.5. The numerical result shows good coincidence with the result from the Buckley-Leverett analysis. Through the numerical simulation by FDM, calculating the pressure equation and the saturation equation subsequently, we can obtain not only the saturation profile but also the pressure profile in the reservoir, which provides useful information for the waterflooding operation in petroleum production technology.

References: 1) Craft, B.C. and Haukins, M., Revised by Ronald E.Terry: Applied Petroleum Reservoir Engineering, Prentice Hall, pp.1-6 (1991). 2) Buckley, S.E. and Leverett, M.C.: Mechanism of Fluid Displacement in Sands, Transactions AIME, Vol.146, pp.107-116 (1942). 3) Aziz,K. and Settari,A.: Petroleum Reservoir Simulation, Applied Science Publishers (1979)