

IMPROVED PREDICTION METHOD OF LATERAL GIRDER RESPONSE OF FOOTBRIDGES

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1. INTRODUCTION

Although cable supported bridges have the structural rationality and elegant feature, they are flexible and often suffer from dynamic problems. The girder vibrated when many people walked on the T-bridge, a cable stayed footbridge in Japan in 1989. The girder vibrated laterally with a frequency of 0.93Hz in the first lateral mode. The mechanism of this vibration was studied by Fujino et al. (1993) and Nakamura et al. (2004, 2006, 2009) and clarified. The gravity center of the human body moves laterally while they walk to their left and right foot in turn, which make the girder to vibrate laterally with a frequency of about 1.0Hz. When the frequency of this dynamic force is close to the bridge natural frequency, it makes a resonance which results in a response. Even though many people walk at the same frequency, their phase would be different and the induced forces could cancel out. This phase problem was solved by assuming that people synchronize their walking pace to the vibrating deck.

The same vibration problem occurred on the London Millennium Bridge in 2000. In order to clarify the mechanism of the vibration, field tests on the bridge and experiments at Imperial College were carried out. Then, a dynamic model based on the single degree of dynamic equation was proposed by Dallard et al. (2001), assuming that the external force induced by pedestrians is proportional to the girder velocity.

Nakamura proposed the simplified model to predict girder responses by introducing non-linear pedestrians' behaviors. When the vibration amplitude becomes large, some of the pedestrians feel unsafe and stop walking or change their walking pace. Therefore, the pedestrian induced forces do not increase proportionally with deck vibration at the large girder amplitude. Nakamura's method includes this non-linear behaviors but it assumes that this lateral vibration only occurs when the bridge natural frequency is around 0.9-1.0Hz. In this paper the improved method is proposed for the bridge natural frequencies. However, the authors have found that the lateral vibration could occur on the bridge with a frequency far less than 0.9Hz. The authors have proposed the new prediction method to cover the resonant frequencies outside 0.9-1.0Hz and the new method is verified by comparing with the field measured data.

2. DYNAMIC MODEL FOR THE LATERAL VIBRATION

The layout of the T-bridge is shown in Fig.1. The T-bridge is a cable-stayed footbridge with a main span length of 134 m, a side span length of 45m, two cable planes and 11 stays per plane (Fig.1) Semi-parallel type cables with epoxy resin surface layer are used. The girder is a steel box girder with orthotropic deck. The web height is 1.8m and the walk way width is 5.26m (Fig.2).

The T-bridge connects the motor boat race stadium and the bus terminal. When the final race was over, the spectators left the stadium and walked through the bridge to get to the bus terminal. At that time the bridge deck was overcrowded with pedestrians. In several minutes of this overcrowded situation the girder and the cables started to vibrate. The field measurements were carried out on the T-bridge to find mechanism of this vibration by Fujino et al. (1993) and Nakamura et al. (2004, 2006 and 2009). Accelerometers were attached to the girder and cables in vertical and lateral directions. By analyzing the obtained data, it was found that the girder vibrated laterally. It was also found that the zigzag movement of pedestrians was the exciting force and the synchronization of the pedestrian to the girder vibration was the phase problem.

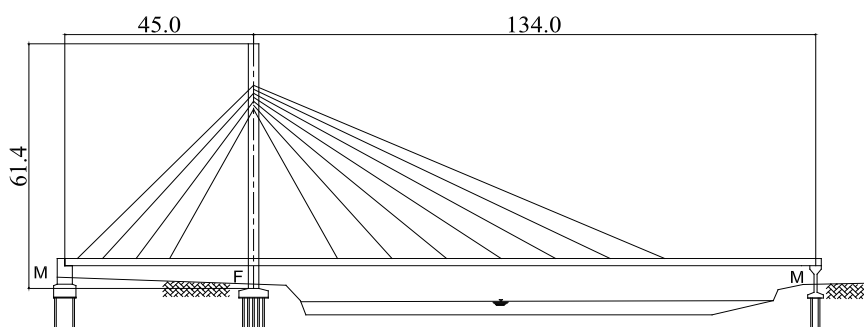


Fig.1 Layout of the bridge (m)

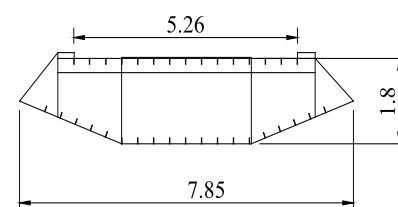


Fig.2 Girder cross-section (m)

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Nakamura (2004) proposed the following method to predict the girder response induced by pedestrians. The dynamic model for the lateral vibration can be modeled by single degree of freedom system. Eq.(1) shows the dynamic equation of motion; X_B is the modal displacement of the girder, X'_B the modal velocity of the girder, X''_B the modal acceleration of the girder, M_B is the modal mass, C_B the modal damping coefficient and K_B the modal stiffness of the bridge.

The external force induced by pedestrians is a key in this dynamic equation. Dallard et al. (2001) proposed that the dynamic force induced by a pedestrian is proportional to the girder velocity. Based on the field test conducted on the Millennium Bridge, the coefficient is proposed to be 300 N.s/m. According to the Dallard method, if the lateral force F_p is larger than the damping force $C_B X'$, there is no upper limit for the girder response and it increases linearly.

This does not seem correct because some pedestrians feel unsafe and hold handrails or stop walking when the vibration amplitude becomes large. Therefore, the exciting force must saturate at a certain level. The right side of Eq. (1) is the external force induced by pedestrians. Nakamura assumed that the pedestrian force is proportional with the modal self-weight of pedestrians, $M_p g$, multiplied by two coefficients, k_1 and k_2 , and two functions, $H(X'_B)$ and $G(f_B)$, as shown in Eq.(2). The coefficient k_1 is the ratio of the lateral force to the pedestrian's weight and is assumed to be 0.0987 in this study. The coefficient k_2 is the percentage of pedestrians who synchronized to the girder vibration and assumed to be 0.2.

$H(X'_B)$ is a function to describe the pedestrian's synchronization nature explained above. The pedestrian forces proposed by Dallard and Nakamura are shown in Fig.3. The one by the Dallard method is linear and no upper limit exists. Whereas, the one by the Nakamura method is linear only in the small girder velocity range and saturates at a certain level. This seems more appropriate to explain the pedestrian's walking attitude on the vibrating deck.

$G(f_B)$ is the function to describe how pedestrians synchronized with the girder response. It was originally thought that the lateral vibration only occurred at the bridge frequency of 0.8-1.0 Hz and, therefore, the value of $G(f_B)$ was assumed to be 1.0, as shown in Eq.(4). This is shown as G_0 in Fig.4. However, the authors have recently found on another bridge that the lateral vibration could occur with the bridge natural frequency outside this range. The authors have proposed the two improved $G(f_B)$ functions, as shown as G_1 and G_2 in Eq. (5) and Eq. (6). Both functions have peaks at 0.9 Hz and Eq. (6) has a sharper shape. These three G-functions are shown in Fig.4. In this study the effect of these new G-functions are compared and studied.

Field measurements were conducted in 1989 and two data were used in this study: Data T1 with a pedestrian density of 0.8 person/m² and data T2 with a pedestrian density of 0.6 person/m². Fig.5 and Fig.6 show the girder lateral displacements of these two cases. Both show clear sinusoidal waves with a natural frequency of 0.93 Hz. This indicates that resonance occurred between the pedestrians and the girder in both cases. The maximum amplitude of T1 is 13.1mm and that of T2 is 8.4mm. These differences are caused by the number of pedestrians, in other words, the pedestrian density. In the T1 data the bridge was more overcrowded than in the T2.

$$M_B X''(t) + C_B X'(t) + K_B X(t) = F_p \dots \dots \dots (1)$$

$$F_p = k_1 k_2 H(X'_B) G(f_B) M_p g \dots \dots \dots (2)$$

$$H(X'_B) = \frac{X'_B(t)}{k_3 + |X'_B(t)|} \dots \dots \dots (3)$$

$$G(f_B) = \begin{cases} G_0 = 1 \dots \dots \dots (4) \\ G_1 = 1 - 20(f_B - 0.9)^4 \dots \dots \dots (5) \\ G_2 = 1 - 5(f_B - 0.9)^2 \dots \dots \dots (6) \end{cases}$$

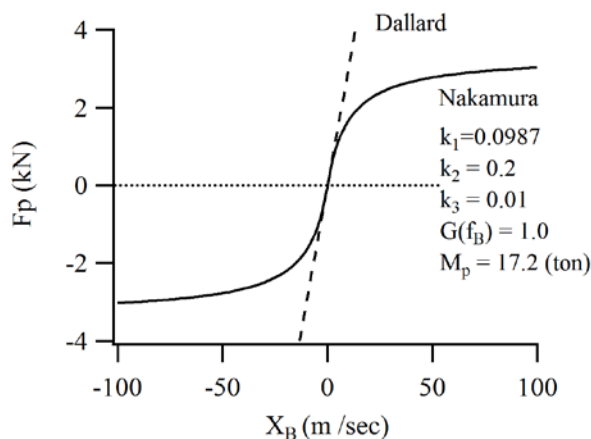


Fig.3 Lateral force induced by pedestrians

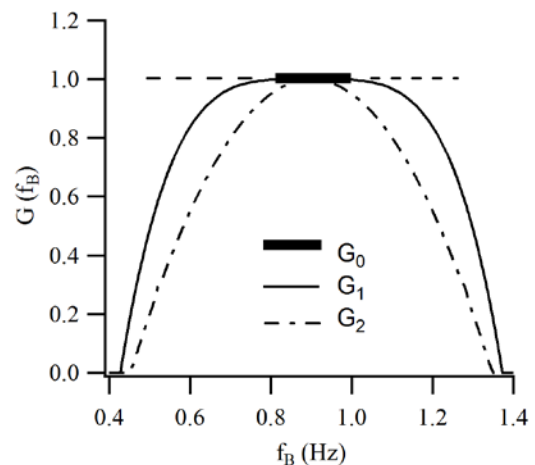


Fig.4 G-function

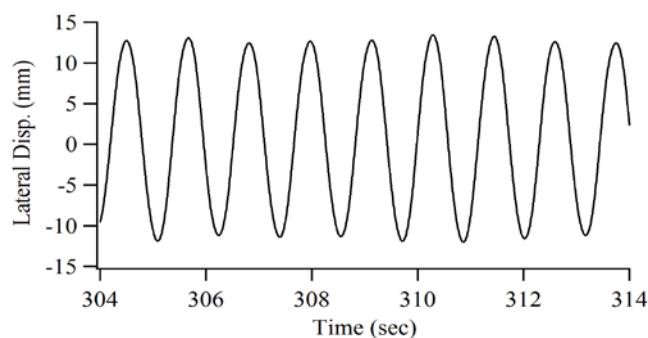


Fig.5 Measured girder displacement (T1)

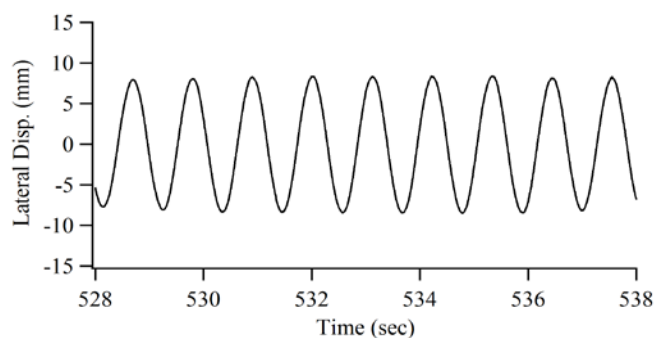


Fig.6 Measured girder displacement (T2)

The eigen-value analysis was conducted by FEM. The natural frequency of the first lateral mode of the T-bridge f_B is 0.93Hz, which is the same as the measured natural frequency of T1 and T2. It is therefore clear that resonance occurred in the first lateral mode. The mode shape is shown in Fig.6. The modal mass M_B is 185.2 ton.

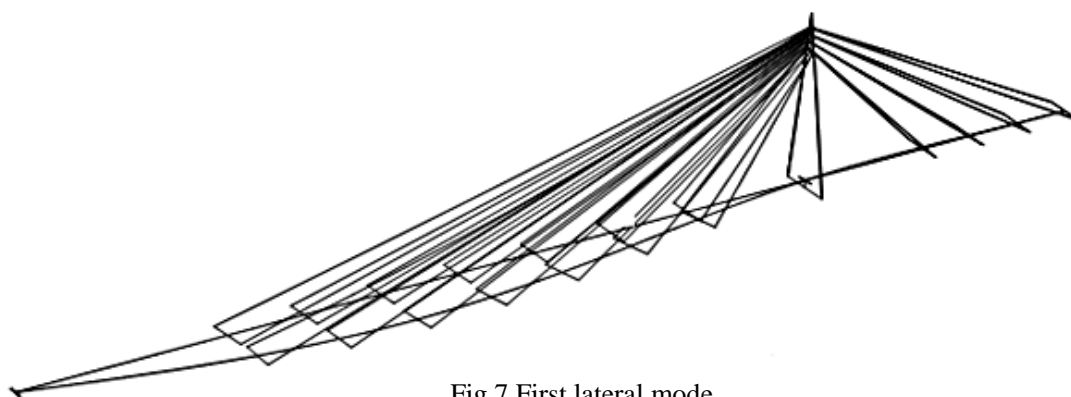


Fig.7 First lateral mode

3. PARAMETRIC STUDIES

The dynamic responses of the girder were obtained for T1 and T2 data by the proposed dynamic model. Eqs. (1)-(6) were solved numerically by Runge-Kutta method (Nakamura et al. 2006, 2009). Parametric studies were conducted to clarify effects of pedestrian density, girder mass and bridge natural frequencies to the girder responses.

Fig.8 shows the girder response with different pedestrian density. In this calculation girder modal mass is kept at 185.2 ton and girder natural frequency of 0.93Hz. The girder response increases with pedestrian density. This tendency is easily understood: the vibration amplitude of the girder becomes larger as the more people walk on the bridge. The measured data, T1 and T2, are also shown in this figure, and agree with the calculated values, which validates the proposed method.

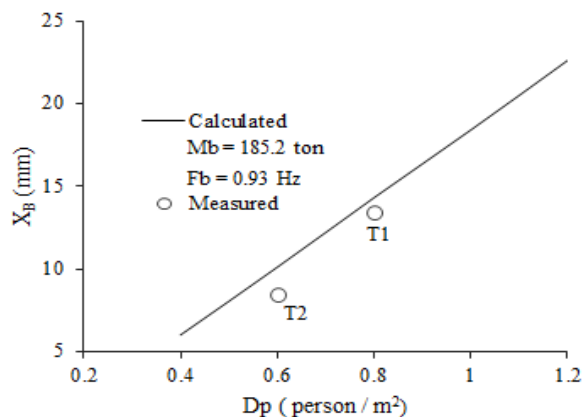


Fig.8 Effect of density of pedestrian

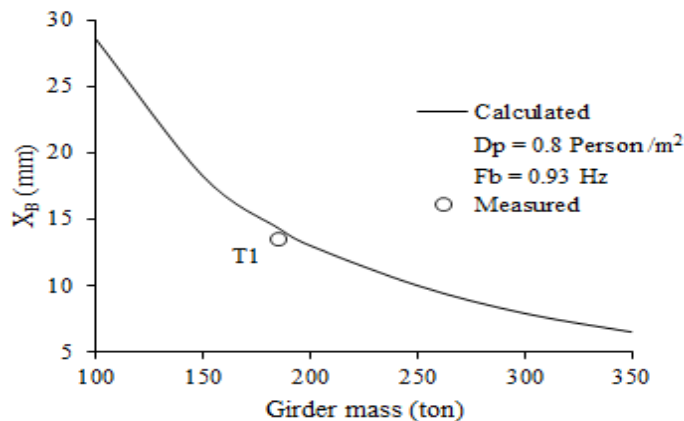


Fig.9 Effect of girder mass

Fig.9 shows the girder response with different girder mass. In this calculation pedestrian density is kept at 0.8 person/m^2 and girder natural frequency of 0.93Hz . The girder response decreases with girder mass, showing that the lighter girder has more likely to have a vibration problem. The measured data T1 is also shown in this figure, corresponding to the calculated value.

Fig.10 shows the girder response ratio; the response calculated by G_1 -function over that by G_0 -function. The response ratio calculated by G_2 -function over that by G_0 -function is also shown in this figure. Both response ratios have a peak at 0.9Hz and they decrease as the bridge natural frequency becomes further away from 0.9Hz . This decreasing ratio with G_2 -function is sharper than that with G_1 -function.

The improved method with G_1 -function and G_2 -function is useful to predict responses when the bridge natural frequency is away from the usual resonant frequency of about 0.9Hz . However, further study is necessary to decide which is more suitable for practical bridges.

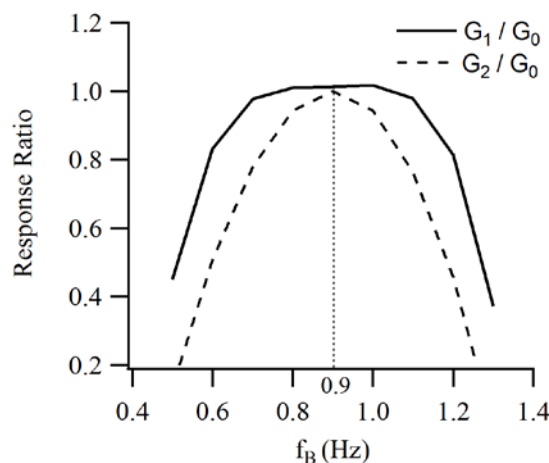


Fig.10 Response ratio due to G_1 and G_2

4. CONCLUSIONS

The lateral vibration problem induced by pedestrian was first observed on the T-bridge and it has then been widely known in the world. This is mainly because many footbridges suffer from this type of vibration because footbridge have become lighter and more flexible. The mechanism of this vibration has almost clarified but the prediction method and the design criteria have not established yet.

Nakamura proposed a simple method to predict response induced by pedestrians which seems useful for practical bridge designers and engineers. One of his assumptions is that this problem only occurs for the bridges with natural frequency of about 0.9Hz ($0.8\text{--}1.0\text{Hz}$). However, this vibration has been observed with natural frequency outside of this range.

This paper has proposed the improved method to cover wide range of bridge natural frequencies by introducing distribution function, G_1 -function and G_2 -function. The predicted girder responses generally agree with the measured values obtained on the T-bridge. Parametric studies were conducted with the proposed method. The girder response increases with pedestrian density. The girder response decreases with girder mass, showing that the lighter girder has more likely to have a vibration problem.

The girder response ratio has a peak at 0.9Hz , and it decreases as the bridge natural frequency becomes further away from this peak frequency. This decreasing ratio with G_2 -function is sharper than that with G_1 -function. The improved method with G_1 and G_2 functions is useful to predict responses when the bridge natural frequency is away from the usual resonant frequency of about 0.9Hz . However, further study is necessary to decide which is more suitable for actual bridges.

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