A numerical simulation model for long-waves shoring by using rectangular grids.

Tokyo Institute of Technology	Student member	Md. Abdul AZIZ
Tokyo Institute of Technology	Member	Keisuke NAKAYAMA
Tokyo Institute of Technology	Student member	Kentaro KUDO
Tokyo Institute of Technology	Fellow	Tadaharu ISHIKAWA

1 INTRODUCTION

The behavior of waves on sloping beaches has received intensive study by many scientists and engineers during the past ten decades. Primary impacts of the behavior of waves are listed as inundation, exacerbation of flooding, beach erosion, and salt-water intrusion to rivers and groundwater aquifers. These impacts, in turn, cause higher-ordered impacts in a wide range of coastal systems. Since there exist highly productive ecosystems, large portion of the world population, and intensive socioeconomic activities in the coastal zone, it is crucial to predict the degree and range of the possible impacts of sea level rise and storm surge inundation in a wide coastal area.

2 MATHEMATICAL MODEL

2.1 Basic equations:

The shallow water equations are used in this model as basic equation.

$$\begin{aligned} \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} &= -g\frac{\partial (h+z_b)}{\partial x} \\ \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} &= -g\frac{\partial (h+z_b)}{\partial y} \\ \frac{\partial h}{\partial t} + \frac{\partial (uh)}{\partial x} + \frac{\partial (vh)}{\partial y} &= 0 \end{aligned} \tag{1}$$

Here, Eq. 1 refers the basic equation of momentum and continuity equation. Where h is the flow depth; u and v indicate flow velocity along x and y direction respectively. Also, g refers the acceleration due to gravity. And then, z_b is the bed elevation.

In this model time splitting is applied for advection phase and non-advection phase. The non-advection phase will be calculated using Galerkin's Finite Element method. Here Eq. 2 shows the time splitting condition and here CIP denotes a computation scheme to calculate advection phase.

$$\begin{aligned} \frac{\tilde{u} - u^n}{\Delta t} &= -g\frac{\partial}{\partial x}(h^n + z_b) \\ \frac{\tilde{v} - v^n}{\Delta t} &= -g\frac{\partial}{\partial y}(h^n + z_b) \\ \frac{\tilde{h} - h^n}{\Delta t} &= -h^n(\partial u/\partial x)^n - h^n(\partial v/\partial y)^n \\ u^{n+1} &= CIP_u(\tilde{u}, \tilde{h}) \\ h^{n+1} &= CIP_h(\tilde{u}, \tilde{v}, \tilde{h}) \end{aligned} \tag{2}$$

2.2 Calculation of Non-advection term:

To reduce the computation time and simplify the matrix, Shape function (ϕ) and Weight function (ψ) are utilized as follows. Figure1 shows those functions.

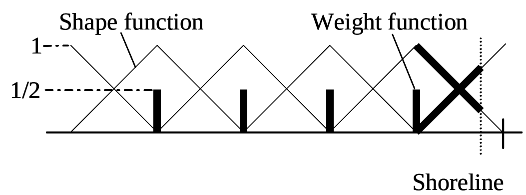

Figure 1 Shape function and weight function.

Furthermore, within the water area, where no boundary exits, the shape and weight functions will be Eq. 3. Figure 2 is showing inner water grid point in two-dimension scheme.

$$\begin{aligned} \psi_i &= \phi_i \\ \phi_1 &= (1-\xi)(1-\eta) \\ \phi_2 &= \xi(1-\eta) \\ \phi_3 &= \xi\eta \\ \phi_4 &= (1-\xi)\eta \end{aligned} \tag{3}$$

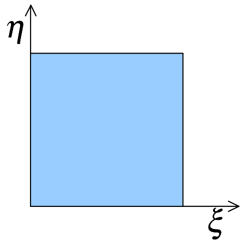

Figure 2 Showing grid point within the water area.

In the two-dimension scheme the grid points at the boundary is shown in Figure 3. The weight functions at different boundary condition are shown in Eq. 4.

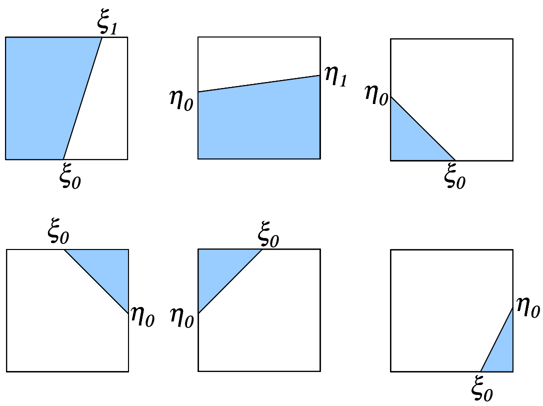

Figure 3 Showing different grid points at boundary.

$$\begin{aligned} \psi_1 &= \partial(\xi)\partial(\eta) \\ \psi_3 &= \partial(1-\xi)\partial(1-\eta) \\ \psi_2 &= \partial(1-\xi)\partial(\eta) \\ \psi_4 &= \partial(\xi)\partial(1-\eta) \end{aligned} \qquad (4)$$

From Eq. 1 applying Galernkin's method, considering left hand side as A1, will give Eq. 5

$$\text{(A1)} \quad \frac{\partial u}{\partial t} \quad \Rightarrow \int_A \dot{U}\phi\psi\, dA\, \Delta x \Delta y \qquad (5)$$

Where $\dot{U} = \frac{\partial u}{\partial t}$

Eq. 6 can be obtained from Eq. 5

$$\begin{bmatrix} A_{11} \ldots\ldots\ldots A_{14} \\ . \\ A_{41} \ldots\ldots\ldots A_{44} \end{bmatrix} \begin{bmatrix} \dot{U}_1 \\ . \\ \dot{U}_4 \end{bmatrix} \Delta x \Delta y \qquad (6)$$

Where $A_{ij} = \int_A \phi_j \psi_i \, dA$

From Eq. 1 and taking right hand side as B1, will give Eq. 7

$$\text{(B1)} \quad -g\frac{\partial h_s}{\partial x} \quad \Rightarrow \int \frac{\partial \phi}{\partial \xi} \dot{H}_s \psi\, dA\, \Delta y \qquad (7)$$

Eq. 8 can be obtained from Eq. 7

$$\begin{bmatrix} B_{11} \ldots\ldots\ldots .B_{14} \\ . \\ B_{41} \ldots\ldots\ldots .B_{44} \end{bmatrix} \begin{bmatrix} \dot{H}_{s1} \\ . \\ \dot{H}_{s4} \end{bmatrix} \Delta y \qquad (8)$$

Where $B_{ij} = \int_A \frac{\partial \phi_j}{\partial \xi} \psi_i \, dA$

From Eq. 1 and taking left hand side of the continuity equation will give Eq. 9

$$\text{(B2)} \quad -g\frac{\partial h_s}{\partial y} \quad \Rightarrow \int \frac{\partial \phi}{\partial \eta} H_s \psi\, dA\, \Delta x \qquad (9)$$

Eq. 10 can be obtained from Eq. 9

$$\begin{bmatrix} B_{111}H_1 + B_{112}H_1 \ldots\ldots\ldots + B_{144}H_4 \\ . \\ B_{411}H_1 + B_{412}H_1 \ldots\ldots\ldots + B_{444}H_4 \end{bmatrix} \Delta x \qquad (10)$$

Where $B_{ijk} = \int_A \frac{\partial \phi_j}{\partial \eta} \psi_i \, dA$

From Eq. 1 and taking horizontal component of the continuity equation will give Eq. 11

$$\text{(B3)} \quad -h\frac{\partial u}{\partial x} \quad \Rightarrow \int_A H\, \dot{U}\, \phi \frac{\partial \phi}{\partial \xi} \psi\, dA\, \Delta y \qquad (11)$$

Eq. 12 can be obtained from Eq. 11

$$\begin{bmatrix} B_{111}H_1 \dot{U}_1 + B_{112}H_1 \dot{U}_2 \ldots\ldots\ldots + B_{144}H_4 \dot{U}_4 \\ . \\ B_{411}H_1 \dot{U}_1 + B_{412}H_1 \dot{U}_2 \ldots\ldots\ldots + B_{444}H_4 \dot{U}_4 \end{bmatrix} \Delta y \qquad (12)$$

Where $B_{ijk} = \int_A \phi_j \frac{\partial \phi_k}{\partial \xi} \psi_i \, dA$

From Eq. 1 and taking vertical component of the continuity equation will give Eq. 13

$$\text{(C1)} \quad -h\frac{\partial v}{\partial y} \Rightarrow \int_A H\, \dot{V}\, \phi \frac{\partial \phi}{\partial \eta} \psi\, dA\, \Delta x \qquad (13)$$

Eq. 13 is the matrix forming of the Eq. 14

$$\begin{bmatrix} C_{111}H_1 \dot{V}_1 + C_{112}H_1 \dot{V}_2 \ldots\ldots\ldots + C_{144}H_4 \dot{V}_4 \\ . \\ C_{411}H_1 \dot{V}_1 + C_{412}H_1 \dot{V}_2 \ldots\ldots\ldots + C_{444}H_4 \dot{V}_4 \end{bmatrix} \Delta x \qquad (14)$$

Where $C_{ijk} = \int_A \phi_j \frac{\partial \phi_k}{\partial \eta} \psi_i \, dA$

In this way after forming the matrix numerical simulation will be done. Results of the numerical simulation will be shown at the time of presentation.

Reference: A new model to solve long waves shoaling on a sloping rectangular grids. K. Nakayama, K. Kudo and T. Ishikawa**.** *Journal of Hydraulic Engineering, JSCE, vol.43,2, 1999*.