(I − 6) FAILURE ANALYSIS OF STRUCTURES USING A NEW SIMPLE TECHNIQUE

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1. Introduction

A new method for failure analysis of structures such as reinforced concrete (RC) and/or steel structures is proposed. For example in case of RC structures, concrete is modeled as an assembly of distinct elements made by dividing the concrete virtually. These elements are connected by distributed springs in both normal and tangential directions. The reinforcement bars are modeled as continuous springs connecting elements together. Local failure of concrete is modeled by failure of springs connecting elements when reaching critical principal stress. The accuracy of the model was checked in the range before rigid body motion starts[1]. This paper introduces a new technique to deal with post failure behavior of structure such as a process of change of structural behavior from continuum state to perfectly discrete state after total failure.

2. Element formulation

We assume that the two elements shown in **Fig. 1** are connected by distributed normal and shear springs at contact points. Each pair of springs fully represent deformation and failure of a certain area. The formulation and results of the element before rigid body motion stage were introduced in Ref. (1) and it was proved that the method could determine deformations and detect the initiation and propagation of cracks. To develop the methodology for post failure analysis, the following steps are proposed.

The general equation of motion is:

$$[M][\Delta\ddot{U}]+[C][\Delta\dot{U}]+[K][\Delta U]=\Delta f(t)+R_m+R_G \quad (1)$$

Where [M] is mass matrix, [C] is damping matrix, [K] is nonlinear stiffness matrix, Δf(t) is incremental applied load vector, [ΔU] is incremental displacement vector and $[\Delta\dot{U}]$ and $[\Delta\ddot{U}]$ are incremental velocity and acceleration vectors, respectively. The term, R_m, is residual force vector due to cracking or incompatibility between strains and stresses at the spring location, while R_G is residual forces due to geometrical changes of the structure during loading. In this technique, we donot have to determine the geometrical stiffness matrix resulting in making the method general and applicable for any case of loading or structure type. The method is applied using the following steps:

1. Assume that R_m and Rg are zeros and solve the equation to get incremental displacement. Newmark Beta method[2] is used for accurate determination of displacements.
2. Calculate incremental and total velocities and accelerations.
3. Modify the geometry of the structure according to the calculated incremental displacements.
4. Modify the direction of spring force vectors according to the new element configuration.
5. Check the situation of cracking and calculate the material residuals load vector R_m.
6. Calculate the element force vector from surrounding springs of each element F_m.
7. Calculate the geometrical residuals around each element from the equation below
$$R_G = f(t) - [M][\ddot{U}] - [C][\dot{U}] - F_m \quad (2)$$
Equation (2) above means that the geometrical residuals account for the incompatibility between external applied forces and internal forces, damping and inertia forces due to the geometrical changes during analysis. Small deformations are assumed during each increment.
8. Calculate the stiffness matrix for the structure in the new configuration considering stiffness changes at each spring location due to cracking or yield of reinforcement.
9. Apply again a new load increment and repeat the whole procedure.

Residuals calculated from the previous increment can be incorporated in solution of **Eq. (1)** to reduce the time of calculation.

3. Numerical results

To check the accuracy of the newly proposed method, large deformation analyses of three case studies are introduced.

The first case is harmonic motion of a bar under its own weight. The bar configuration and results are shown in **Fig. 2**. Two different initial excitation angles were used (θ_o=0.05 and 0.3 rad). The result of small excitation angle was compared with the theoretical one. The calculated X-displacement is almost the same as the one obtained from theoretical kinematics.

The second case is also harmonic motion of a "L" shaped bar under its own weight. The bar configuration and results are shown in **Fig. 3**. It can be shown that the bar starts oscillation around the stability position. Oscillation reduces gradually and finally stops at the equilibrium position. The simulated angle of final stability is the same as the calculated one from theory.

Those two analyses show that the rigid body motion of the structure can be simulated and the final equilibrium position can also be reached.

The third example shows the time history of failure process of a single bay RC frame. The frame is supported by hinged bearing (left) and hinged roller bearing (right). A concentrated load is applied at the center of the beam of the frame. The frame shape, dimensions, loading conditions and deformations under the applied load are shown in **Figs. 4** and **5**. The failure process can be summarized as follows:

1. Cracking starts from the center of the beam.
2. Reinforcement bars yield in the center of the beam.
3. Steel bars cut off after yield.
4. Refering to **Fig. 5**, displacements drastically increase after 0.7 seconds because of failure of reinforcement bars. At the same time, the structure begins unstable dynamic motion.
5. Tension cracks appear at the left connection first, because of the rigid body motion restriction caused by the hinge.
6. Tension cracks appear at the right connection together with motion of the roller.
7. The structural members lose curvature and moves as three rigid bodies in the space.

4. Conclusions

In this study, a new technique was developed by which structure behavior can be followed during loading till complete failure. The failure process can be simulated for elastic region, nonlinear region, and even after separation of structural members. At this moment, the main limitation of the model is that the collision effects are not taken into account. However, research is ongoing to consider the collision effects in the model.

References

1) Meguro K. and Tagel-Din H.: A new efficient technique for fracture analysis of structures, Bulletin of Earthquake Resistant Structure, No. 30, pp. 103-116, 1997.
2) Chopra K.: Dynamics of structures, theory and applications to earthquake engineering, Prentice Hall, 1995.

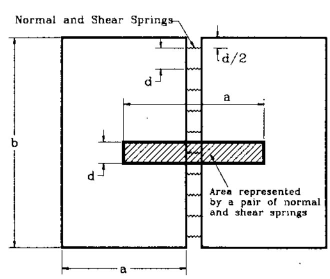

Fig. 1 Spring distributions and area of influence of each pair of springs

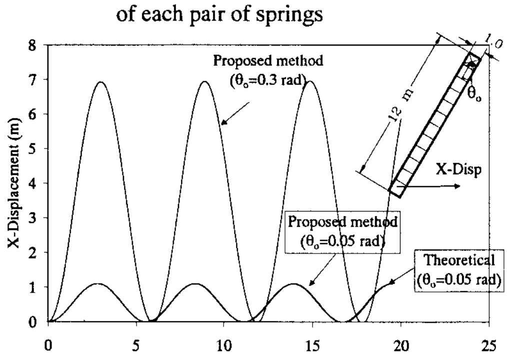

Fig. 2 Harmonic motion of a rigid bar under own weight and initial excitation.

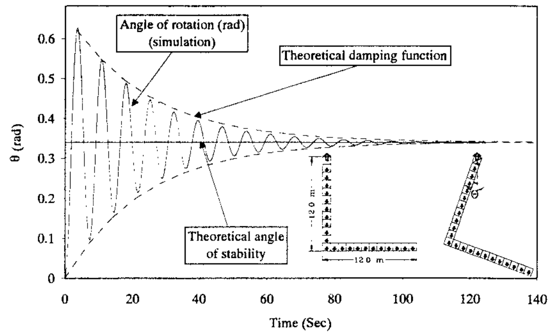

Fig. 3 Harmonic motion of a rigid "L" bar under its own weight. Damping ratio is 4%.

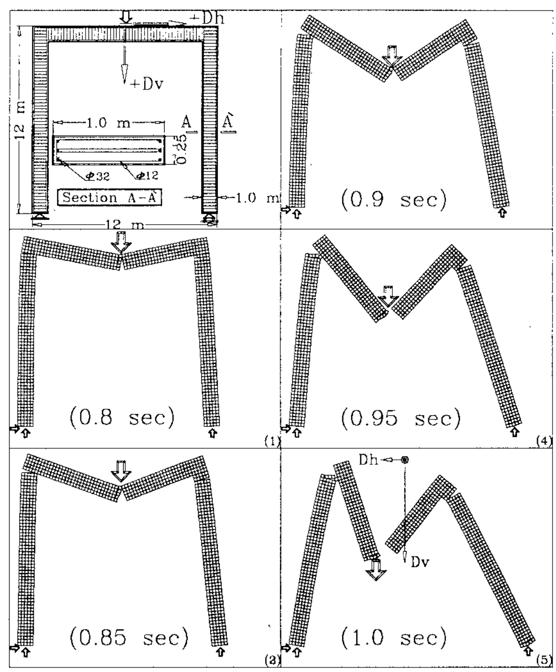

Fig. 4 Deformed shape and failure pattern of a hinged-roller RC frame

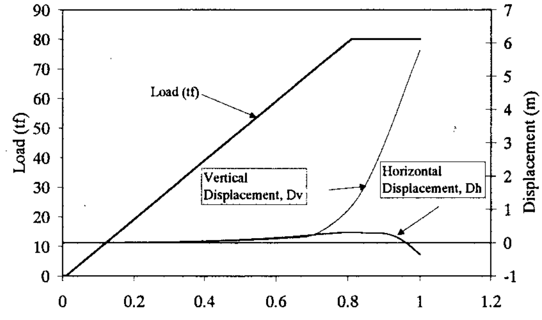

Fig. 5 Load, vertical and horizontal displacement at the loading point vs. time