(I −13) **Analysis of an Embedded Circular Arch: Hydrostatically Distributed Horizontal Loading**

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1. Introduction

Exploration of the ocean resources has become a common engineering project. As the site goes farther away from land, more time and energy will be required for mobilization. However, if a quasi-permanent shelter is constructed on the site itself, time and energy may be optimized. This shelter may be enclosed by an underwater dome composed of, or supported by, intersecting circular arches. The same dome can, likewise, serve as an enclosure to underground housing structures in line with the objective of optimizing land space.

This paper, therefore, is aimed at analyzing the basic structural member of the aforementioned dome, i.e., the embedded arch. The vertical loading condition has been treated in refs. (1) and (2) for Case 1 (uniformly distributed load) and Case 2 (circularly varying load). For this paper, hydrostatically distributed horizontal loading (Fig. 1), subdivided into Case 3 (uniformly distributed horizontal loading, Fig. 2) and Case 4 (triangularly distributed horizontal loading, Fig. 3), is considered. Analysis is done using the Castigliano's theorem which can be expressed in equation form as $\delta = \partial U/\partial P$, where δ is the displacement; U is the total strain energy due to moment, shear and axial forces, and P is the load in the direction of the displacement. These expressions are given in refs.(1) and (2).

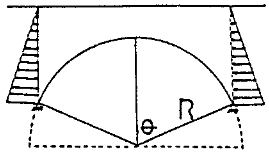


Fig. 1 Hydrostatically Distributed Horizontal Load

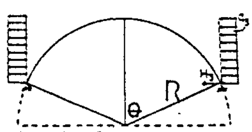


Fig. 2 Uniformly Distributed Loading (Case 3)

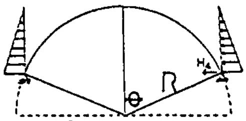


Fig. 3 Triangularly Distributed Loading (Case 4)

2. Horizontal Redundant Reaction.

Using the general expression for the horizontal redundant reaction at the right support as given in refs. (1) and (2), the corresponding reactions for cases 3 and 4, respectively, are:

$H3 = -[q3R^2/(6K)]\{[R^2/(2EI)][\,8\sin\theta + \sin\theta\cos\theta(40\cos\theta - 36) + \theta(12 - 30\cos\theta + 24\cos^2\theta - 36\cos^3\theta) + 18\cos^2\theta(1 - \cos^2\theta)/\sin\theta] + [1/(GFA)][\sin\theta(3 + 4\sin^2\theta) + 3\sin\theta\cos^2\theta - 6\theta] + [1/(AE)][\sin\theta(12 - 4\sin^2\theta) + \sin\theta\cos\theta(6 - 12\cos\theta) + \theta(6 - 12\cos\theta)]\} + q3R(1 - \cos\theta)$, where $q3 = \gamma Db$, and

$H4 = -[2R^2/(KEI)]\{\theta[-2.5k3\cos\theta - 1.875k3 + 2Mc\cos\theta] + \sin\theta\cos\theta[-0.5k3\cos\theta + 1.791666667k3 + (1/12)k3\cos^2\theta] + \sin\theta[3k3 - 2Mc]\} - [2R/(KGFA)]\{\theta[0.625k4] + \sin\theta\cos\theta[(2/3)k4\cos\theta - 0.375k4 - 0.25k4\cos^2\theta] + \sin\theta[(-2/3)k4] - k2 + \cos\theta[k2\cos\theta]\} + [2R/(KAE)]\{\theta[0.875k4] + \sin\theta\cos\theta[(-2/3)k4\cos\theta + 0.875k4 + 0.25k4\cos^2\theta] + \sin\theta[(-4/3)k4]\} - k1$, where $k1 = \gamma bR^2(1 - \cos\theta)^2/2$; $k2 = k1(1 - \cos\theta)/(6\sin\theta)$; $k3 = \gamma bR^3/6$; $k4 = \gamma bR^2/2$; $k5 = -k1 - H4$, $Mc = k2R\sin\theta$, and K is given in refs. (1) and (2).

A numerical analysis for a submerged arch has been done using the following numerical data: R=radius of arch=10 m; γ=specific weight of surrounding medium =9810 N/m^3; E=modulus of elasticity= 2X10^{11}; b=cross-sectional width=0.15m; t=cross-sectional thickness=0.20 m; F=shape factor=5/6; G=shear modulus of elasticity = 7.612X10 N/m^2, and D=depth of embedment=40 m. Distribution for the horizontal redundant reaction for any arbitrary support angle θ for cases 3 and 4 are shown in Figs. 4 and 5, respectively.

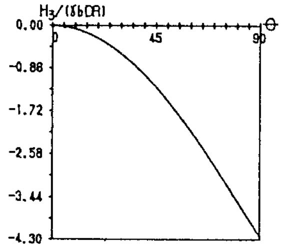


Fig. 4 Distribution of H_3 (Case 3)

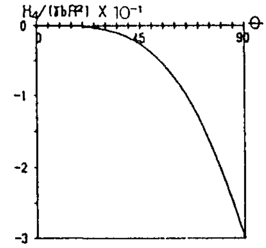
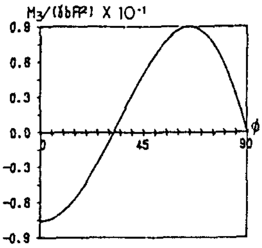


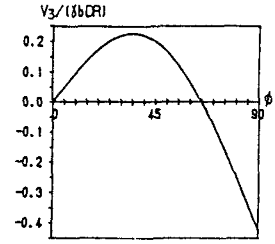
Fig. 5 Distribution of H_4 (Case 4)

3. Moment, Shear and Axial Forces

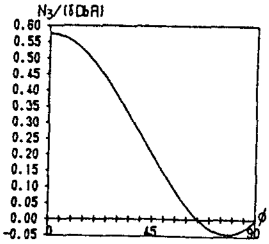
3.1 Case 3. The moment, shear and axial forces at any arbitrary point on the arch axis can be determined, respectively, as $M_\phi = -(R/2)[q_3R\cos^2\phi + (2H_3 - 2q_3R\cos\theta)\cos\phi + (q_3R\cos^2\theta - 2H_3\cos\theta)]$; $V_\phi = (H_3 - q_3R\cos\theta)\sin\phi + q_3R\sin\phi\cos\phi$, and $N_\phi = (H_3 - q_3R\cos\theta)\cos\phi + q_3R\cos^2\phi$. The corresponding distributions for a 90° arch with data as given in the preceding section are given in Figs. 6, 7 and 8.



Moment Distribution
(Case 3, θ=90°)
Fig. 6

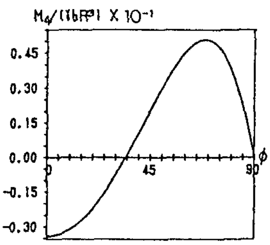


Shear Distribution
(Case 3, θ=90°)
Fig. 7

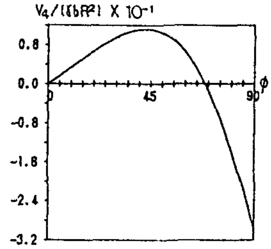


Axial Force Distribution
(Case 3, θ=90°)
Fig. 8

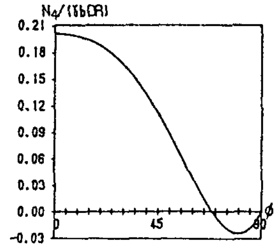
3.2 Case 4. Due to case 4 loading, $M_\phi = M_c - R(1 - \cos\phi)(P_\phi/3 + k_5)$; $V_\phi = -P_\phi \sin\phi - k_5 \sin\phi$, and $N_\phi = -P_\phi \cos\phi - k_5 \cos\phi$, where $P_\phi = k_4(1 - 2\cos\phi + \cos^2\phi)$; $M_c = -R(1 - \cos\theta)[(2/3)k_1 + H_4]$, k_1, k_4 & k_5 are given in section 2. The corresponding distributions are shown in Figs. 9, 10 and 11.



Moment Distribution
(Case 4, θ=90°)
Fig. 9



Shear Distribution
(Case 4, θ=90°)
Fig. 10



Axial Force Distribution
(Case 4, θ=90°)
Fig. 11

4. Displacements

After combining both horizontal and vertical deformations, the final deformed shapes for cases 3 and 4 and exaggerated correspondingly are shown in Figs. 12 and 13, respectively.

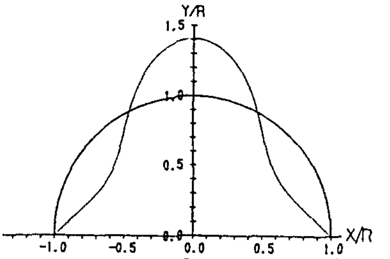


Fig. 12 Final Deformed Shape Exaggerated 10X (Case 3, θ=90°)

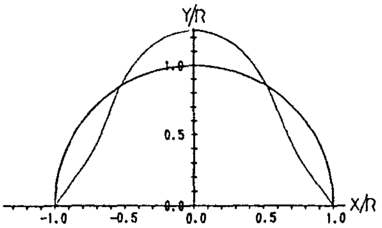


Fig. 13 Final Deformed Shape Exaggerated 50X (Case 4, θ=90°)

6. Conclusion. Using the classical Castigliano's theorem, analysis of an embedded arch due to hydrostatically distributed horizontal loading has been presented. Numerical graphs for the distribution of internal forces and displacements have been shown. However, a dynamic analysis considering wave forces and ground vibration will be considered in the very near future.

7. References

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