

# Preliminary study on the effects of fiber dispersion characteristics on the tensile strength of UHPFRC based on micromechanics

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## 1. INTRODUCTION

Ultra-High Performance Fiber Reinforced Concrete (UHPFRC) is fiber-reinforced composite material with excellent mechanical properties. The general compressive strength and tensile strengths of UHPFRC are over 150MPa and 9MPa, respectively. Besides, due to a high density, the UHPFRC exhibited a very low water permeability providing it a superior durability<sup>1)</sup>. In addition, since no coarse aggregates and rebars are used in UHPFRC, structures can be designed with smaller thickness than ordinary reinforcement structures.

However, due to the excellent flowability of UHPFRC and the exploited heavy steel fibers, the fibers may flow with the matrix and sink. As a result, instead of 3-dimensional completely random as generally cognized, there are certain regulations for the fiber distribution and orientation which may influence the mechanical properties of UHPFRC.

Therefore, the purpose of this study is to investigate the effects of the regulations induced by the flowability on the tensile strength of UHPFRC theoretically from the debonding and sliding of a single fiber by setting certain constrains to the probability density function of the distribution of the fiber's angle. In addition, parametric studies on the fiber volume fraction are conducted to evaluate the effect of the possible fiber sink during casting before hardening.

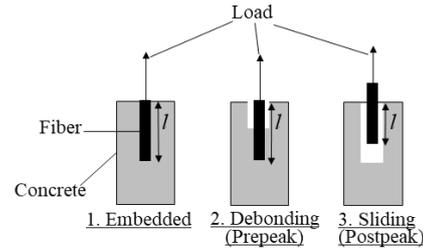
## 2. METHOD<sup>2)</sup>

For a two-side single fiber, the three stages of the pullout of the shorter side are shown in **Fig.1**.  $l$  is shorter half-length of fiber. Firstly, fiber-matrix interface bonded elastically is stretched. Secondly, interface debonding initiates from the loading end and gradually spreads to the embedded end in the debonding stage. Finally, the entire debonded fiber is sided out in sliding stage.

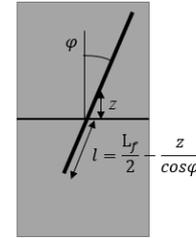
In the debonding stage, the pullout load-crack opening width relation is expressed as

$$P(\delta) = \pi \sqrt{\frac{E_f d_f^3 \tau \delta}{4}} \quad \delta \leq \delta_0 \quad (1)$$

where  $E_f$  is elasticity modulus of fibers,  $d_f$  is fiber diameter,  $\tau$  is constant frictional bond and  $\delta$  is opening width.  $\delta_0 = 2l^2\tau/E_f d_f$  is width of the opening when full debonding occurs. In this study, the values of  $E_f$ ,  $d_f$ , and  $\tau$  are set as 200000 MPa, 0.16mm, and 1 MPa, respectively. In the sliding stage, the pullout load-crack opening width relation



**Fig.1** Two-side single fiber pullout behavior



**Fig.2** Illustration of  $z$  and  $\varphi$

expressed as

$$P(\delta) = \pi t l d_f \left[ 1 - \frac{(\delta - \delta_0)}{l} \right] \quad \delta_0 < \delta \leq l \quad (2)$$

Besides, for the two-side single fiber, the location and orientation with respect the crack face can be determined by  $z$  and inclined angle  $\varphi$  as show in **Fig.2**. Correspondingly, the load function of embedded angle is expressed as Eq. (3) considering a snubbing effect.

$$P(\delta, l, \varphi) = P(\delta, l) e^{f\varphi} \quad (3)$$

where  $f$  is snubbing coefficient.

Next, the location and orientation vary relative to the crack surface. To represent the distribution of the fibers, probability density functions of  $z$  and  $\varphi$  are derived. Firstly, as the distribution of  $z$  from the cracked surface to the center of the fiber is uniform, the probability density function can be expressed as

$$p(z) = \frac{2}{L_f}, \quad 0 \leq z \leq \frac{L_f}{2} \quad (4)$$

where  $L_f$  is whole length of one fiber. In this study,  $L_f$  is set as 13 mm. On the other hand, due to the high flowability of UHPFRC, the fiber orientation may exhibit a tendency alignment to the flow direction. Specifically, the range of  $\varphi$  is expected to be smaller than  $(0, \pi/2)$ . Thus, by setting the upper limit of  $\varphi$  to  $\varphi_0$ , the probability density function is

calculated and given as

$$p(\varphi) = \frac{\sin\varphi}{1 - \cos\varphi_0} \quad (0 \leq \varphi \leq \varphi_0) \quad (5)$$

Finally, the prepeak and postpeak bridging stress-crack opening width relation was derived by substituting equation (1) or (2), (3), (4) and (5) to (6).

$$\tilde{\sigma}_B = V_f \int_{\varphi=0}^{\varphi_0} \int_{z=0}^{\left(\frac{L_f}{2}\right)\cos\varphi} \frac{P(\delta, l, \varphi)}{\pi \left(\frac{d_f}{2}\right)^2} p(\varphi)p(z) dz d\varphi \quad (6)$$

where  $V_f$  is fiber volume fraction.  $\tilde{\sigma}_B = \sigma_B/\sigma_0$  and  $\sigma_0 = V_f(L_f/d_f)/2$ . After normalization and simplification by ignoring minor components, the prepeak and postpeak bridging stress-crack opening width relations can be expressed as

$$\tilde{\sigma}_B = \left[ 4 \left( \frac{\tilde{\delta}}{\tilde{\delta}^*} \right) - 2 \frac{\tilde{\delta}}{\tilde{\delta}^*} \right] \int_0^{\varphi_0} e^{f\varphi} p(\varphi) \cos\varphi d\varphi \quad (7)$$

for  $0 < \tilde{\delta} < \tilde{\delta}^*$

$$\tilde{\sigma}_B = 2(1 - \tilde{\delta})^2 \int_0^{\varphi_0} e^{f\varphi} p(\varphi) \cos\varphi d\varphi \quad (8)$$

for  $\tilde{\delta}^* < \tilde{\delta} < 1$

where  $\tilde{\delta} = \delta/(L_f/2)$  and  $\tilde{\delta}^* = \delta_0/(L_f/2)$  which are normalized by the half of the fiber length.

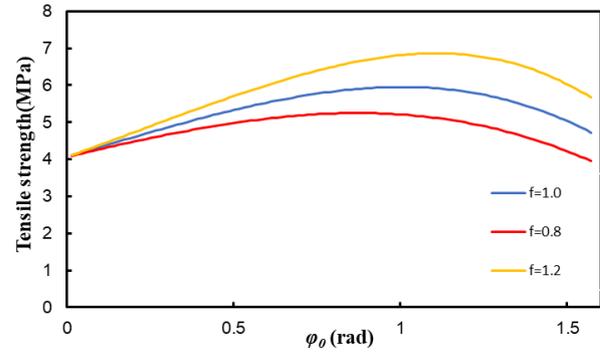
### 3.RESULT AND DISCUSSION

This study focuses on investigating the tensile strengths along the direction of the matrix flow. **Fig.3** shows the tensile strength -  $\varphi_0$  relationships for the snubbing coefficient  $f$  equaling 0.8, 1.0, and 1.2 when  $V_f$  is fixed to be 5%. It is found that the maximum tensile strength appears at different  $\varphi_0$  for different  $f$ . This may be understood as follows. When  $\varphi_0$  increases, the proportion of fibers with large inclined angle increases resulting in more significant snubbing effect. Nevertheless, the resistance to fiber pull-out may be decreased due to the large inclined angle at the same time. Under the combined effects from these two contributors, a  $\varphi_0$  between 0 and  $\pi/2$  gives the maximum tensile strength, and the  $\varphi_0$  with the maximum tensile strength increases with the increasing of the snubbing coefficient. Besides, the change of tensile strength related to the  $\varphi_0$  is more remarkable for higher snubbing coefficient. Thus, the fiber alignment due to flowability may not always leads to a higher tensile strength, whereas the effects of fiber alignment may be more apparent for stiff type of fibers like steel fibers in UHPFRC.

To evaluate the effect of sink of heavy fibers, the variation of tensile strength induced by the fiber volume fraction  $V_f$  is analyzed as well, where different  $V_f$  is to simulate different degree of fiber sediment. **Table 1** shows calculated tensile strengths of  $V_f$  equaling 3%, 5% and 7% for several  $\varphi_0$ , where  $f$  is set as 1.0. It is found that for each  $\varphi_0$  the tensile strength increases monotonically with the increasing and the  $V_f$  induced variation is more apparent than that of  $\varphi_0$ , which means that the fiber sink should be paid high attention and appropriately controlled by exploiting rapid hardening composites using heavy reinforcing fibers.

**Table 1** Tensile strength -  $V_f$  relations

	Tensile strength (MPa)		
	$V_f=3\%$	$V_f=5\%$	$V_f=7\%$
$\varphi_0 = \pi/2$	2.83	4.72	6.61
$\varphi_0 = \pi/3$	3.49	5.82	8.14
$\varphi_0 = \pi/6$	3.22	5.37	7.52
$\varphi_0 = \pi/8$	3.05	5.09	7.12



**Fig.3** Embedded angle-tensile strength relations

### 4.CONCLUSION

The effects of two fiber dispersion characteristics, i.e. fiber flow and sink, on the tensile strength of UHPFRC were evaluated numerically based on micromechanics, where the two characteristics were simulated by limiting the variation range of the inclined angle of the fibers relative to the crack surface and the fiber volume fraction. It was found that the fiber sink may lead to a monotonic variation of the tensile strength. Nevertheless, the fiber alignment induced by flow may not always lead to tensile strength increasing as the snubbing effect may decrease at the same time.

As this study is a simple simulation by arbitrarily changing the parameters representing fiber dispersion characteristics, the next step may be determining the parameters according to experimental investigations of the fiber characteristics of real specimens as well as predicting the mechanical properties. In addition, due to the fiber flow, it may be unreliable to treat the UHPFRC as an isotropic material. Thus, material properties along other directions should also be investigated.

### ACKNOWLEDGEMENT

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