Fatigue analysis of an RC bridge slab subjected to a stepwise loading sequence using the bridging stress degradation concept

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1. INTRODUCTION

Reinforced concrete (RC) bridge slab, one of the most prone structural members to fatigue, is typically subjected to a large number of load repetitions from heavy traffic. It is reported to deteriorate significantly due to the continuous increase in traffic amount and insufficient slab thickness in the past design. Therefore, many experimental and numerical studies on the prediction of fatigue life of RC bridge slab have been conducted. According to previous experimental investigations, the fatigue life of RC slab under moving load is lesser than that of slab under fixed pulsating load [1, 2].

Experimental and numerical studies on fatigue durability evaluation for highway bridge slabs were carried out by Public Works Research Institute (PWRI). In the numerical method, the damage accumulation theory (miner's rule) was employed and stiffness of the damaged concrete elements was reduced. The numerical method evaluated the fatigue durability with some degree of reliability [3].

Li and Matsumoto[4] introduced bridging stress degradation concept in fatigue crack growth analysis of fiber reinforced concrete. Later, this concept was used in numerical modelling to predict the fatigue behavior of RC slabs and it efficiently predicted fatigue life of RC slabs under moving and fixed pulsating loads[5]. The applicability of the bridging stress degradation concept was extended to predict the fatigue life of the RC slabs reinforced with plain bars [6]. These studies confirm the validity of the numerical model based on bridging stress degradation for RC slabs subjected to constant loading, but the stepwise loading sequence is introduced in the improved specification of the road bridge for fatigue durability of slab. Thus, the applicability of this numerical model for RC slab subjected to stepwise loading sequence has to be investigated.

In this study, fatigue analysis of RC bridge slab subjected to stepwise loading sequence based on the bridging stress degradation concept is carried out, and the results are compared with those of experimental and numerical method of PWRI. The developed numerical model based on bridging stress degradation concept is showing good agreement with the experimental results.

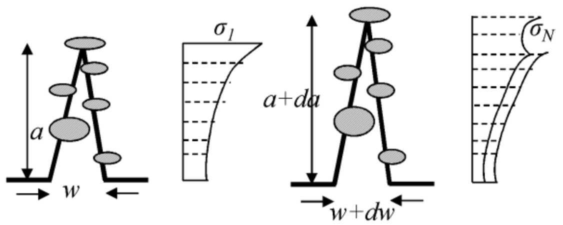
2. METHOD

A finite element method (FEM) based on Newton-Raphson iteration technique is applied to solve the nonlinear behavior of the concrete. Nonlinear constitutive laws of concrete in tension and compression are shown in **Table 1** [7]. The stress-strain relation of concrete in tension is considered linear until

Table 1 Nonlinear constitutive laws of concrete

Compression		Tension*	
$0 \ge \varepsilon \ge \varepsilon_m$	$\sigma = f_c \dfrac{\varepsilon}{\varepsilon_m}\left(2 - \dfrac{\varepsilon}{\varepsilon_m}\right)$	$\varepsilon_t \ge \varepsilon \ge 0$	$\sigma = E_c \varepsilon$
$\varepsilon_m \ge \varepsilon \ge \varepsilon_u$	$\sigma = f_c \dfrac{\varepsilon_u - \varepsilon}{\varepsilon_u - \varepsilon_m}$	$\varepsilon \ge \varepsilon_t$	$\sigma = f_t \left(\dfrac{\varepsilon_t}{\varepsilon}\right)^{0.4}$

E_c = modulus of elasticity of concrete,
f_c' = concrete compressive strength,
ε_m $= f_c/2E_c$ =concrete strain corresponding of f_c'
f_t = tensile strength and $\varepsilon_t = f_t/E_c$ = strain at tensile strength
*The multiplier 0.4 can be used for the concrete



(a)　1^{st} cycle　　　(b)　N^{th} cycle

Figure 1 Crack propagation due to bridging stress: **(a)** 1^{st} cycle **(b)** after N^{th} cycles

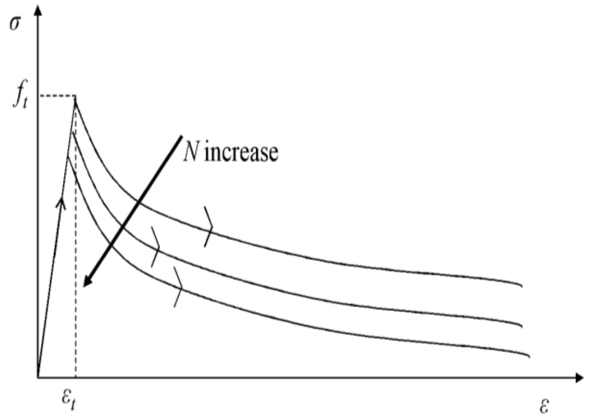


Figure 2 Hysteric behavior of concrete under cyclic loading

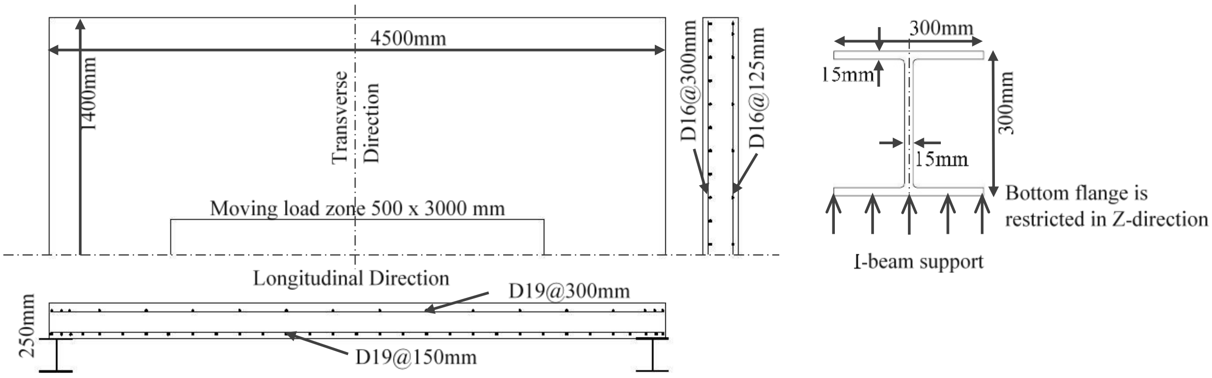


Figure 3 RC slab geometry

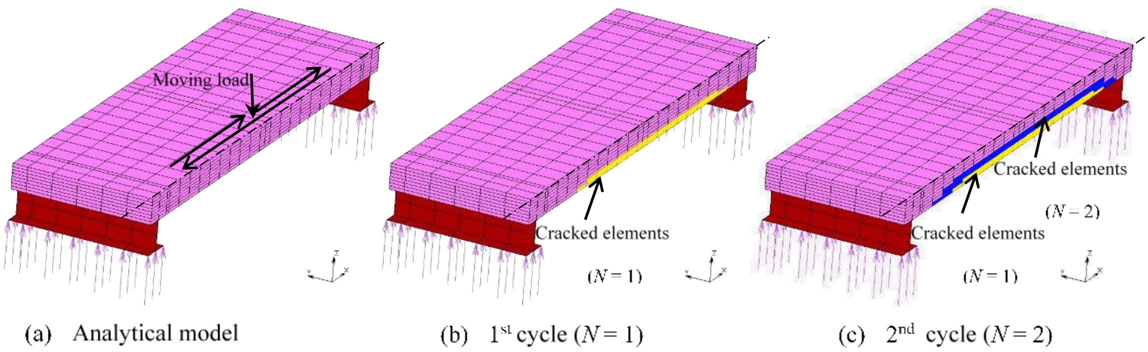


Figure 4 Analytical procedure

cracking. After initiation of cracking, the crack bridging model is employed for representation of concrete behavior in tension.

According to bridging stress degradation concept, a crack begins with length (a) and width (w) due to first loading as displayed in **Figure 1(a)**. The crack passes through a process of opening and closing resulting in the reduction of bridging stress in repetitive moving loads. Thus, the existing crack grows with increased length (da) and increased width (dw) as shown in **Figure 1(b)**. This crack propagation and reduction in bridging stress is termed as bridging stress degradation majorly depending upon number of cycles (N) and maximum tensile strain (ε_t) [3]. The bridging stress degradation equation obtained from uniaxial fatigue results is employed in this analysis as follows [8]:

$$\frac{\sigma_N}{\sigma_1} = 1-\left(0.08+4\times\delta_{\max}\right)\log(N) \tag{1}$$

where δ_{max} is modified for smeared crack elements in the form of tensile strain (ε_t) and smeared crack element size (l) as $\delta_{max}=\varepsilon_t \times l$. The stress strain behavior of concrete in tension under cyclic loading is shown in **Figure 2**. No plastic strain is considered due to same loading and reloading path.

For stress-strain relationship of reinforcement bar, a bilinear curve with yield stress (f_y) is used. The Giuffré-Menegotto-Pinto model [9] is utilised to represent the hysteretic behavior of reinforcement bar under repetitive loading as following:

$$\frac{\sigma}{f^*_y} = H\frac{\varepsilon}{\varepsilon^*_y} + \frac{(1-H)\dfrac{\varepsilon}{\varepsilon^*_y}}{\left[1+\left(\dfrac{\varepsilon}{\varepsilon^*_y}\right)^R\right]^{1/R}} \tag{2}$$

$$R = R_o - \frac{a_1\xi_{\max}}{a_2-\xi_{\max}} \tag{3}$$

where H is hardening parameter, R_o, and R are transition parameters between elastic hardening for the 1^{st} and N^{th} cycle (R_o = 20), $\zeta_{\max}$ is the maximum excursion in the plastic range, a_1 and a_2 are the parameters for change of R with repetitive load history (a_1 = 18.5 and a_2 = 0.00015).

A RC slab with dimensions 4500×2800×250 mm is used in this study. Dimensions and reinforcement details of the tested RC slab are shown in **Figure 3**. The same experimental properties of concrete and reinforcement bars are used in this analysis as shown in **Table 2**.

Table 2 Material properties

Material	Strength (MPa)	Modulus of Elasticity (MPa)
Concrete	f_c' = 32.4	21727
Steel	f_y = 345	200000

The moving load zone is 3000×500 and the load is applied along the longitudinal direction. The loading type is stepwise loading sequence; the initial load is 157 kN and increased in step with increase in no. of cycles as shown in **Figure 5**.

The numerical model is a three-dimensional finite element method (FEM) based on smeared crack elements representing

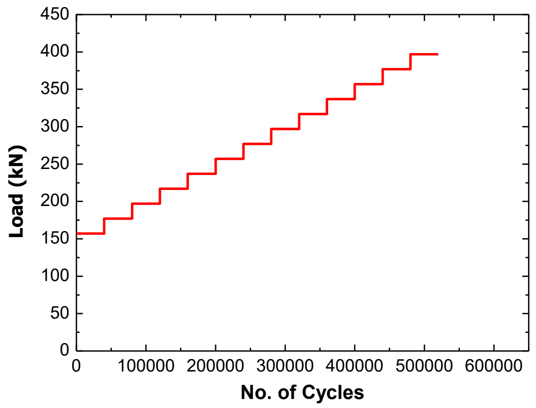


Figure 5 Stepwise loading sequence

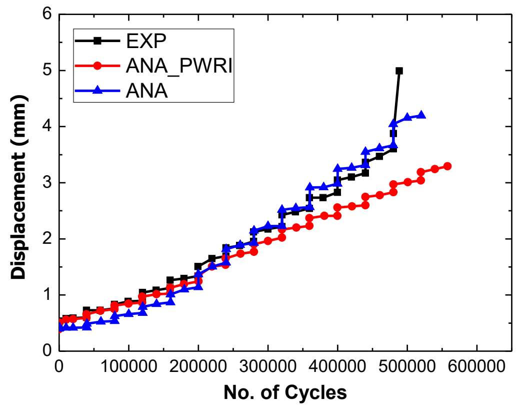


Figure 6 Centre displacement evolution

concrete and reinforcement bars. Concrete cracks are permitted to begin in three perpendicular directions in each crack element according to the principal stress cracking criteria.

A finite element software MSC/MARC is used to model the slab, a 3D model using solid elements, as displayed in **Figure 4(a)**. Due to symmetry in loading and boundary conditions, a half of the slab is analysed. Firstly, the moving load is applied at the central elements of the slab. Then, these elements are unloaded while the other elements, adjacent to the right side of the loaded elements, are loaded simultaneously. In this technique, a load is made to move along the longitudinal direction. This loading leads to the propagation of cracked elements in the first cycle as shown in **Figure 4(b)**. According to the crack bridging stress degradation concept, the constitutive law for the cracked elements is modified. After the second cycle of moving load, new cracked elements are appeared as displayed in **Figure 4(c)**. The overall RC slab stiffness is decreased with the increase of cycles of moving load due to bridging stress degradation and crack propagation. The procedure is continued until fatigue failure occurred and the numerical results are recorded in each cycle of moving load.

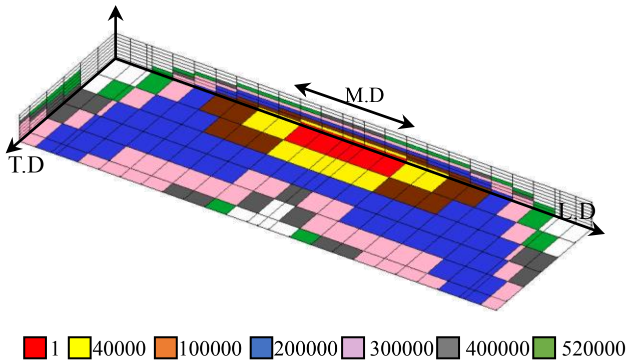
3. RESULTS AND DISCUSSION

3.1 Centre displacement evolution

The displacements at centre of the RC slab obtained during the fatigue analysis at different loading cycles in this study and numerical study by PWRI are compared with displacements observed in experiment as shown in **Figure 6**.

In the numerical method by PWRI, the damage accumulation theory (miner's rule) is employed and the cumulative damage degree of the concrete element after analysis step is calculated. If cumulative damage degree reaches 1, the concrete element is considered to be cracked and elastic modulus of the cracked concrete element is reduced to 1/10th of original elastic modulus. The two extreme stages are considered whether the concrete element is undamaged or damaged completely. In the reality, the stiffness of concrete element gradually reduces due to fatigue phenomenon. The approach of treating the element between healthy and damaged resulted into smaller displacements as compared to experimental ones.

In this current numerical model based on the bridging stress degradation concept, the bridging stress degradation equation is introduced after initiation of cracking. After each analysis step, the reduction of bridging stress in concrete element is employed majorly depending upon number of cycles (N) and maximum tensile strain (ε_t). Also, cracks are permitted to begin in three perpendicular directions in each cracked element according to the principal stress cracking criteria. This numerical model is predicting smaller centre displacements at initial cycles as compared to those of experimental, but it well accurately predicts the similar centre displacements to experimental ones after initial cycles of moving load. The reason of the accurate displacement prediction is that the numerical model is able to capture the dominant degradation mechanism. However, the degradation mechanism may not be so dominant in the initial cycles.



M.D: Moving Direction　　L.D: Longitudinal Direction
T.D: Transverse Direction

Figure 7 Propagation of cracked elements

3.2 Propagation of cracked element

This numerical model considers primary aspect that the propagation of cracked elements and degradation of bridging stress are the core reasons of the fatigue failure. Thus, it is significant to show the propagation of cracked elements. The uncracked elements are shown by white color and the cracked

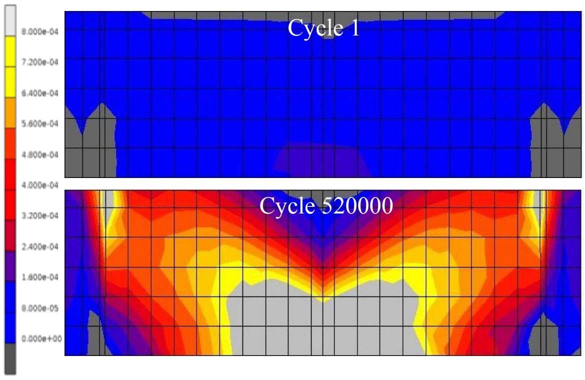


Figure 8 Maximum principal strain distribution

elements after first cycle of moving load are displayed by red color. Similarly, the cracked elements caused by different number of cycles are indicated with different colors as shown in **Figure 7**. From the figure, it can be observed that the cracked elements are spread all over the longitudinal direction of RC slab, whereas the cracked elements are distributed from mid span of RC slab towards transverse direction with less degradation. Since the moving load effect is transferred along the longitudinal direction, the cracked zone is larger in the longitudinal direction as compared to the transverse direction.

The comparison of maximum principal strain distribution on bottom surface of RC slab after 1st cycle and last cycle is displayed in **Figure 8**. The strain is increased in diagonal direction from loading point towards the supporting corner. The increase in tensile strain in diagonal direction provides justification of propagation of cracked elements in diagonal direction as shown in **Figure 7**.

3.3 Cracking pattern

The cracking pattern on bottom surface of an RC slab under moving load at fatigue failure predicted by the numerical model based on the bridging stress degradation concept is examined as shown in **Figure 9**. The main crack is started at the centre, the first location of the moving load, thereafter extends to the supporting corners. As the load begins to move, diagonal cracks are formed between the locations of the moving load in the longitudinal direction and the supporting corner, making the first crack set. At the same time, other cracks are formed perpendicular to the existing cracks as the second and third crack sets, surrounding the moving load zone to create grid crack. An increase in the number of cycles resulted in much more extensive cracking.

4. CONCLUSIONS

In this paper, the applicability of numerical model based on the bridging stress degradation concept for an RC bridge slab subjected to stepwise loading sequence is investigated. The centre displacement evolutions of RC bridge slab obtained in this study and numerical study by PWRI are compared with the experimental one.

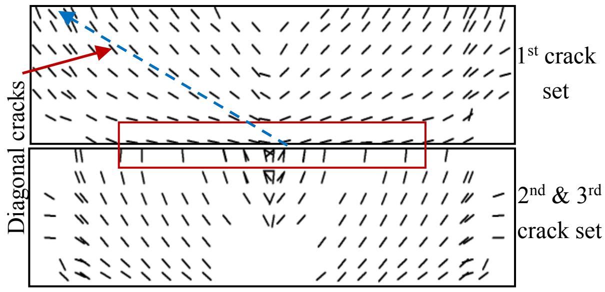


Figure 9 Cracking pattern on bottom surface

The developed numerical model based on the bridging stress degradation concept for RC bridge slab subjected to stepwise loading sequence is showing good agreement with the experimental results.

The numerical model is able to predict the crack pattern in RC bridge slab subjected to stepwise loading sequence

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