THEORETICAL MODELING OF ALTERNATE BAR FORMATION INCORPORATING BANK EROSION

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1. INTRODUCTION

Depending upon the fluid flow and sediment transport conditions, a variety of structures can develop on the river bed whenever perturbations are induced. These include ripples, dunes, antidunes and alternate bars. While the first three types of bedform provide a significant resistance to the flow by the river bed, control the river depth, and are periodically formed [1], alternate bars can exist together with dunes and are often formed as isolated features. These megascale bedforms arise upon the order of the channel width and sediment size.

Alternate bars can cause river banks to erode; the river banks themselves contribute to the deformation of the channel planform through bank erosion. In this context, alternate bars allow for the onset of meandering [1]. Meandering is considered to be a process of exchange of sediments from the caving banks to the depositing bars, and from the consequent local overloading and deposition of the heavier sediments as they move along the bed[2]. Fully-developed meandering requires a considerable balance between erosion and deposition from the bed and the banks [1].

Fig. 1. Alternate bars in the Naka River, an artificially straightened river in Japan. Image courtesy: S. Ikeda. Available: slideplayer.com. [5]

Alternate bar studies can help provide relevant information on their hydraulic characteristics, such as their formative conditions, geometrical properties, and migration velocities[3] that can aid in river restoration measures. These measures may include providing vegetation, increasing cohesion to control bank erosion, and any other means of river bank stabilization.

The initial development of alternate bars can be modeled analytically by using linear stability analysis. From the analysis, the phase shift between the perturbed bed and other perturbed parameters such as the flow velocities, sediment transport rates and the banks. The phase shift is important for instability and can describe the relation of one perturbed parameter to the other.

We perform linear stability analysis using the standard equations for flow and sediment motion including the process-based bank erosion model by Parker et al.[4] A simple 3D bed topography showing the growth of alternate bars is presented. Finally, the phase shift is obtained to show the location of the perturbed bed with respect to the perturbed bank. The phase shift is also used to briefly explain the relation between the lateral sediment transport rates with respect to the erosion and deposition of the banks and the bed.

2. FORMULATION

The governing equations used are the two-dimensional shallow water equations and the Exner equation for sediment continuity. These equations in non-dimensionalized form are:

$$U\frac{\partial U}{\partial x}+V\frac{\partial U}{\partial y}=-\frac{1}{F^2}\left(\frac{\partial H}{\partial x}+\frac{\partial Z}{\partial x}\right)-\beta\frac{T_{bx}}{H} \tag{1}$$

$$U\frac{\partial V}{\partial x}+V\frac{\partial V}{\partial y}=-\frac{1}{F^2}\left(\frac{\partial H}{\partial y}+\frac{\partial Z}{\partial y}\right)-\beta\frac{T_{by}}{H} \tag{2}$$

$$\frac{\partial UH}{\partial x}+\frac{\partial VH}{\partial y}=0 \tag{3}$$

$$\frac{\partial Z}{\partial t}+\frac{\partial Q_{bx}}{\partial x}+\frac{\partial Q_{by}}{\partial y}=0 \tag{4}$$

where (U,V) are the flow velocity components in the x and y directions, respectively; H is the flow depth, Z is the bed elevation, F is the unperturbed Froude number, C_f is the bed friction coefficient taken as constant and equal to 0.01, and β is the aspect ratio. The aspect ratio is the ratio of the channel width to the flow depth.

The boundary conditions used are the vanishing lateral velocity at the sidewalls, the exchange of sediment from the banks to the bed, and the migration rates or process-based bank erosion model by Parker et al.

3. RESULTS AND DISCUSSION

The higher the aspect ratio, the higher the instability and increase of bar mode, m. It is then expected that alternate bars (m = 1) develop at low aspect ratios. Higher aspect ratios may show bar modes greater than 1.0 and a tendency to braid; this is beyond the context of the study.

The simple 3D bed model is plotted at $\beta = 10$, as shown in Fig. 2. It can be shown that the bar mode, m, is equal to 1.0, which means that alternate bars form.

The bed topography shows the alternating riffles and pools of the channel. Although the illustrated figure is plotted for a relatively short streamwise length, the objective is to confirm that at a low aspect ratio, alternate bars occur.

Since the illustration in Fig. 2 only shows the bed topography, the location of the perturbed banks cannot be shown and the effect of bank erosion could not be explained by Fig. 2. In this regard, the bed perturbation is compared to the bank perturbation using phase difference.

As shown in Fig. 3, the amplitude of the bed and bank perturbations are plotted against the streamwise direction. The right and left banks are in-phase. Since the coordinate of the right bank is y = 1/2, the crest of the bank waveform is taken as eroding, and the trough is depositing. In the same way, since the bed has a positive amplitude, the crest is the high bed elevation, and the trough the low bed evelation. Fig. 3 shows that the bed leads the right bank (i.e. the bed reaches its amplitude first than the right bank). This may be attributed to the fact that the bed process is assumed to evolve first than the bank process.

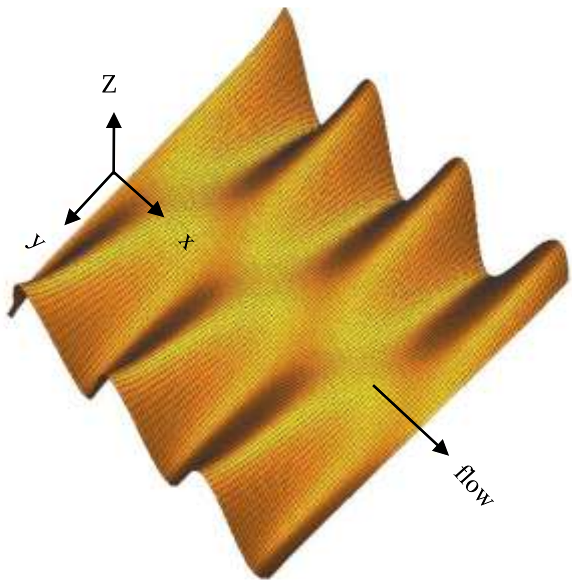

Fig. 2 Bed topography showing initial development of alternate bars. ($F = 0.2,\ \theta_n = 0.06,\ \theta_c = 0.05,\ S_R = S_L = 1.73,\ k = 1.2,\ \beta = 10,\ m = 1$)

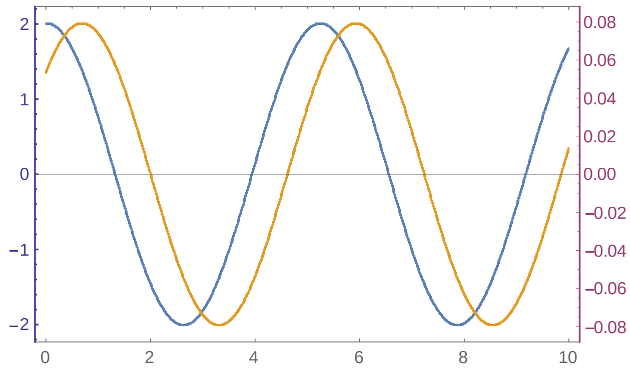

Fig. 3 Phase difference between bed (blue) and right bank (orange) perturbations. ($k = 1.2,\ \beta = 10,\ bank\ slopes = 1.73$)

In addition, the most eroded part of the bank does not coincide with the deepest part of the bed. It is possible that under the given flow and sediment conditions, the phase lag of the bank with respect to the bed is due to the lateral migration of the channel evaluated at the junctions. As erosion causes the adjacent bed to be scoured, the bed is consequently lowered. From the bank erosion model, original junction is shifted outward. When the banks slowly migrate, the location of the eroded part of the bank is being shifted away from the deepest part of the bed.

In the same manner, the phase difference between the lateral sediment delivery rate from the right bank to the bed, qyRj, and the right bank perturbation, is shown in Fig.4. The waveforms are completely out-of-phase, with only a relatively small phase shift angle. As mentioned, since the right bank is evaluated at $y = 1/2$, the crest of the waveform is taken as the eroded part, while the trough is the deposited part. For qyRj, the crest is taken as eroding, while the trough is depositing. Considering the origin $x = 0$, it is shown that as the lateral sediment transport rate is increasing, the right bank amplitude is decreasing. Also, if the perturbation is to be followed in the streamwise direction, as the lateral sediment transport rate is decreasing, the right bank perturbation is increasing. It can be interpreted that as more sediment is eroded from the bank than is deposited, the right bank is eroded, and as more sediment is deposited to the bank than is being entrained by the flow, the right bank is absorbing sediment. When the rate of sediment delivered from the bed is high, erosion occurs, and the bank is eroded. On the other hand, when the delivery rate is low, deposition occurs, and the bank is absorbing the sediment and is being deposited.

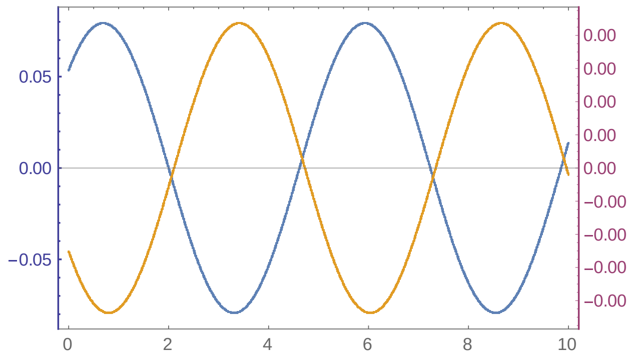

Fig. 4 Phase difference between right bank (blue) and qyRj (orange) perturbations. ($k = 1.2,\ \beta = 10,\ bank\ slopes = 1.73$)

4. CONCLUSION

Using linear stability analysis, the initial development of alternate bars is studied analytically. The effect of bank erosion is clarified. The relation of the lateral sediment transport rates to the evolution of the bed and right bank is also investigated through phase difference. The study is able to provide a simple bed topography showing the formation of alternate bars and incipient meandering. Finally, in obtaining phase difference, the lateral migration of the channel causes the shift of the location of the bed with respect to the bank, whereby the deepest part of the bed does not coincide with the most eroded part of the bank.

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