Fatigue Analysis of Frost Damaged RC Slabs Subjected to Moving Load Based on Bridging Stress Degradation Concept

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1. INTRODUCTION

Concrete as a porous material has the ability to absorb andretain moisture. This characteristic makes unprotected concrete structures susceptible to frost damage.The typical frost damage is surface scaling (characterized by concrete surface loss) and damage in material microstructure that is manifested macroscopically by irreversible tensile deformation and randomly oriented microcracking.

Frost damage is one of the main damage to reinforced concrete (RC) deck slab in cold regions. RC deck slabs with frost damage, in service, are subjected to repetitive loadings. This fact shows that we need to know the behaviour of frost damaged RC deck slabs under fatigue loadings.

Previous studies focused on determining the mechanical properties of frost damaged concrete under static and fatigue loadings. Hasan et al.[1] investigated the dependence of mechanical behaviour of concrete such as strength, stiffness, and deformation capacity on damage caused by freezing and thawing cycle (FTC). A significant reduction in strength and stiffness in both compression and tensionwas observed. The FTC parameter was introduced to explain the degradation in initial strength and stiffness of concrete resulting from freezing and thawing damage. However, no study has been found on the behaviour of frost damaged RC deck slabs under fatigue loadings.

Most of the existing studies deal with fatigue analysis of RC deck slabs without frost damage consideration. Different methods are used. The recently developed fatigue analysis method is proposed for solving the problems of concrete structures depending on the general concept that the propagation characteristics of both flexural and shear cracks are taken into account as an inducement of the failure. The bridging stress degradation relation is proposed for cementitious materials such as plain concrete and fiber concrete. The concept of fatigue crack propagation was proposed for predicting fatigue life of fiber reinforced concrete beams by Li and Matsumoto[2] and RC beams by Suthiwarapirak et al.[4]. The bridging stress degradation, the reduction of transferred stress across a crack under fatigue loading,has been introduced for the first time by Li and Matsumoto[2] as a principal cause of fatigue crack propagation in concrete and fiber reinforced concrete beams.

In the present study, a three-dimensional model of FEM is proposed for predicting the fatigue performances of frost damaged RC deck slabs. The fatigue life and crack elements propagation, will be numerically obtained by this study and will be compared with experiments.

2. METHOD

2.1. Concrete model

In this study, the following laws have been adopted as concrete model as shown in **Fig.1**. In compression, when the strain ε is within ε_m (concrete strain corresponding to concrete compressive strength f_c) to 0, the compressive stress σ and the concrete tangential modulus E are defined respectively by equations 1 and 2; and when $\varepsilon < \varepsilon_m$, the compressive stress σ and the concrete tangential modulus E are defined respectively by equations 3 and 4.

In tension, linear elasticity is used before cracking, i.e. until tensilestress of concrete reaches the concrete tensile strengthf_t. After cracking ($\varepsilon > \varepsilon_t$= strain at tensile strength), equations5 and 6 respectively express the tensile stress and the concrete tangential modulus.

Equations for compression

$$\sigma = f_c \frac{\varepsilon}{\varepsilon_m}\left(2 - \frac{\varepsilon}{\varepsilon_m}\right) \tag{1}$$

$$E = 2\frac{f_c}{\varepsilon_m}\left(1 - \frac{\varepsilon}{\varepsilon_m}\right) \tag{2}$$

$$\sigma = \frac{f_c \cdot \varepsilon_u}{\varepsilon_u - \varepsilon_m} + E \cdot \varepsilon \tag{3}$$

$$E = \frac{f_c}{\varepsilon_m - \varepsilon_u} \tag{4}$$

Equations for tension

$$\sigma = f_t\left(\frac{\varepsilon_t}{\varepsilon}\right)^{0.4} \tag{5}$$

$$E = -0.4 \cdot \frac{f_t}{\varepsilon} \cdot \left(\frac{\varepsilon_t}{\varepsilon}\right)^{0.4} \tag{6}$$

2.2. Bridging stress degradation

The fatigue model used is based on bridging stress degradation concept (**Fig. 2**). The bridging stress is the transferred stress across a crack through aggregates.Under repetitive loading a crack is subject to closing and opening process. This process leads to a reduction of bridging stress due to progressive deterioration of bond between aggregates. The reduction of the bridging stress causes a stress concentration at the crack tip resulting in crack propagation. The degradation law of concrete is a function of two parameters: maximum tensile strain ε_{tmax}, and number of cycles N and is defined as follows[2,3]:

$$\frac{\sigma_N}{\sigma_1} = f(N, \varepsilon_t) = 1 - (0.08 + 4\varepsilon_{tmax} l) log(N) \tag{7}$$

where l is cracked element size. σ_N and σ_1 are bridging stress at the Nth and the first cycle, respectively.

2.3. Frost damaged concrete model

Stress-strain model of frost damaged concrete by Hasan et al[1] (**Fig.3**) is used to take into account the reduction of the concrete strength and stiffness caused by frost damage. This model, based on plasticity and fracture of concrete elements, introduced the FTC fracture parameter to explain the degradation in initial strength and stiffness of concrete resulting from freezing and thawing damage. Based on experimental data, the FTC fracture parameter was empirically formulated as a function of plastic tensile strain caused by freezing and thawing with the assumption that the plastic strain is caused by the combined effects of FTC and mechanical loading damage. The stress-strain relationship can be obtained from

$$S = \alpha\beta K_0 C_0 (E - E_p) \tag{8}$$

$$\alpha = e^{-1.70E_{pf}} e^{1.70E_{pf}^{0.15}E_{max}^{0.85}} \quad for E_{max} < E_{pf} \tag{9}$$

$$\alpha = 1 \;\; for \;\; E_{max} \geq E_{pf} \tag{10}$$

$$\beta = e^{-0.45E_{pf}(1-e^{-30E_{pf}})} \tag{11}$$

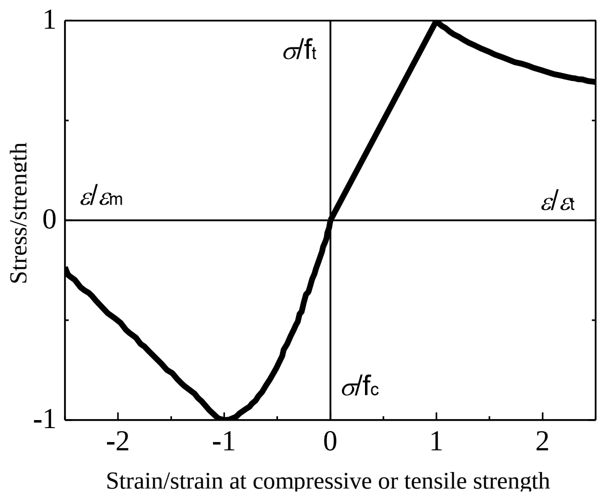

Fig. 1 Normalized stress-strain relationship for concrete

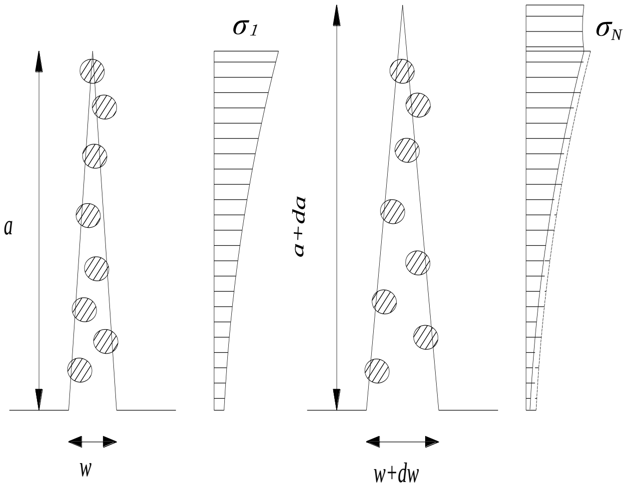

Fig. 2 Bridging stress degradation

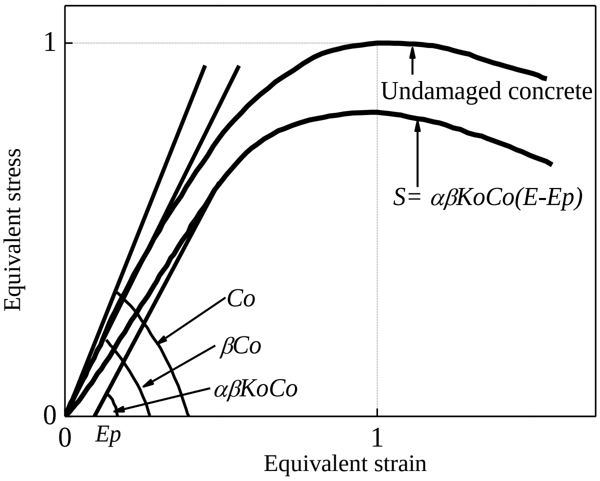

Fig. 3 Equivalent stress-equivalent strain relationship for frost damaged concrete[1)]

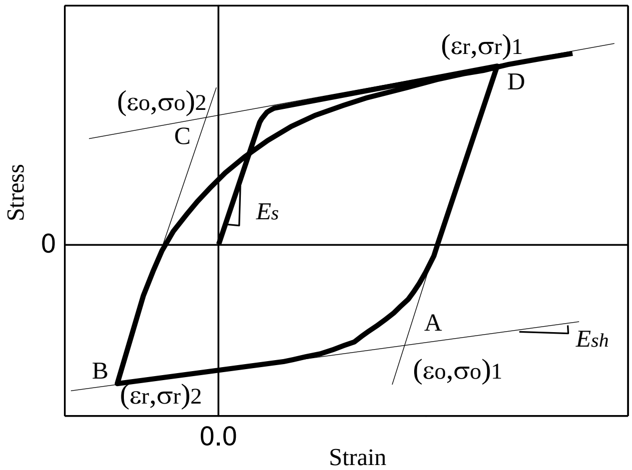

Fig. 4 Giuffré-Menegotto-Pinto model illustration[6)]

$$E_p = E_{max} - a(1 - e^{-bE_{max}}) \tag{12}$$

$$K_o = e^{-0.73E_{max}(1-e^{-1.25E_{max}})} \tag{13}$$

$$a = \frac{20}{7} - 2.10E_{pf} + 0.34E_{pf}^2 \tag{14}$$

$$b = 0.35 - 0.25E_{pf} + 0.18E_{pf}^2 \tag{15}$$

where S, α, β, K_0, C_0, E, E_p, E_{pf}, E_{max} are respectively equivalent stress,effective factor, FTC fracture parameter, fracture parameter, initial stiffness, equivalent strain, equivalent plastic strain, FTC equivalent plastic strain, and maximum equivalent strain.

2.4. Reinforcing bar model

The stress-strain behaviour of a rebar under repetitive load is simulated in this study by using Giuffré-Menegotto-Pinto model[5)]. As illustrated in **Fig. 4**, the model is described by a smooth curve asymptotic to the tangent lines at the point of stress reversal and at the point of maximum/minimum strain in the loading history. Mathematical equations representing the model are given in terms of the normalized stress-strain relation as

$$\sigma^* = b\varepsilon^* + \frac{(1-b)\varepsilon^*}{(1+\varepsilon^{*R})^{1/R}} \tag{16}$$

$$\varepsilon^* = \frac{\varepsilon-\varepsilon_r}{\varepsilon_o-\varepsilon_r} \tag{17}$$

$$\sigma^* = \frac{\sigma-\sigma_r}{\sigma_o-\sigma_r} \tag{18}$$

$$R = R_o - \frac{a_1}{a_2+\xi}\xi \tag{19}$$

where $\sigma^*$and ε^* are normalized stress and strain, (ε_o, σ_o) is the intersection point of two tangents (points A, C in **Fig. 4**), (ε_r, σ_r) is the unloading/reloading point (points B, D in **Fig. 4**). b,R, R_o, $\xi$are respectively strain hardening ratio which is defined as the ratio of the tangent of the curve at the maximum/minimum point in loading history (E_{sh} in **Fig. 4**) to that at the unloading/reloading point (E_s in **Fig. 4**), parameter defining the shape of transitioncurve between elastic and hardening, value of R during first loading, the strain difference between the interception point (ε_o, σ_o) and the point of maximum/minimum in the loading history, a_1 and a_2 are parameter for the change of R with cyclic loading history.

2.5. Analytical procedure

Finite element method is used to solve a slab model of smeared crack elements. A moving load of dimension 120 x 300 mm is applied along the slab length centerline as indicated in **Table 1**. Under the action of this load cracked elements are formed. As the number of cycles increases, cracked elements are modified according to the bridging stress degradation conc-

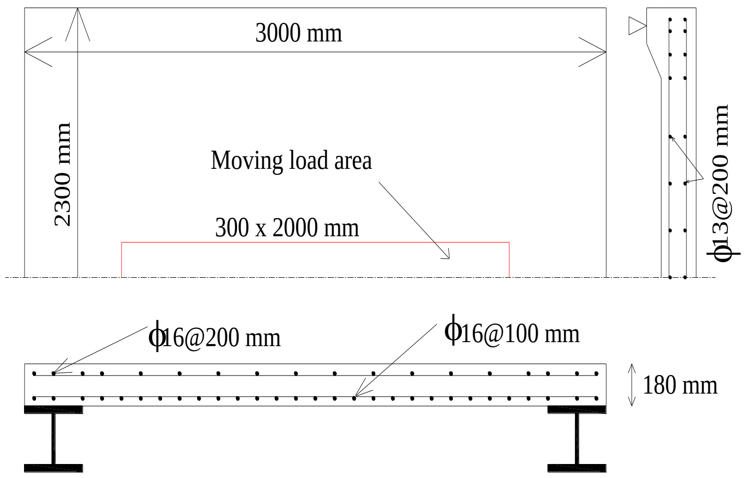

Fig. 5 Slab geometry

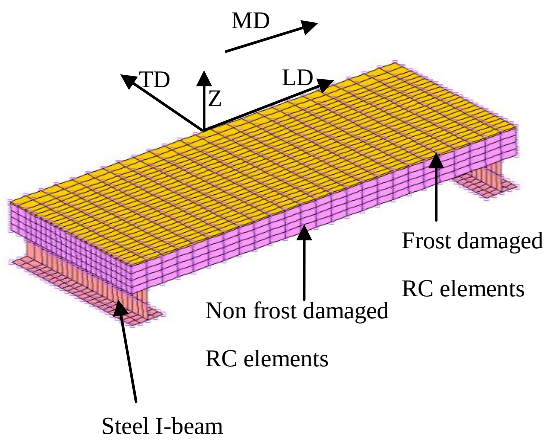

Fig. 6 Analytical model

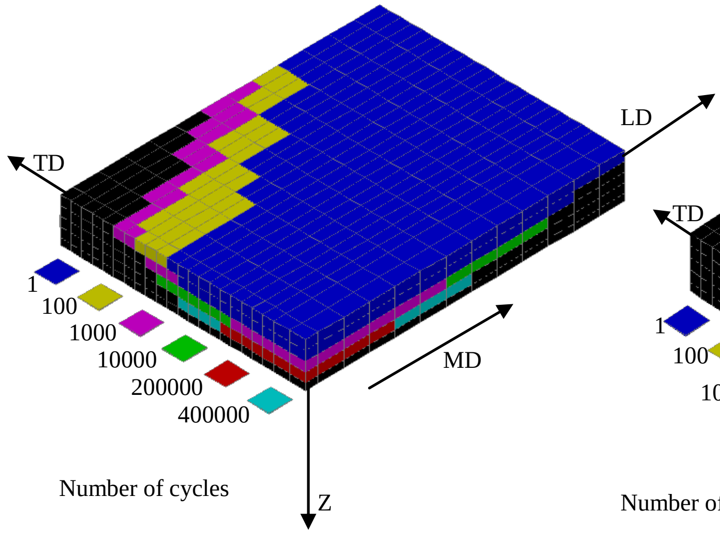

Fig.7 Propagation of cracked elements for slab without frost damage (bottom surface showed on top)

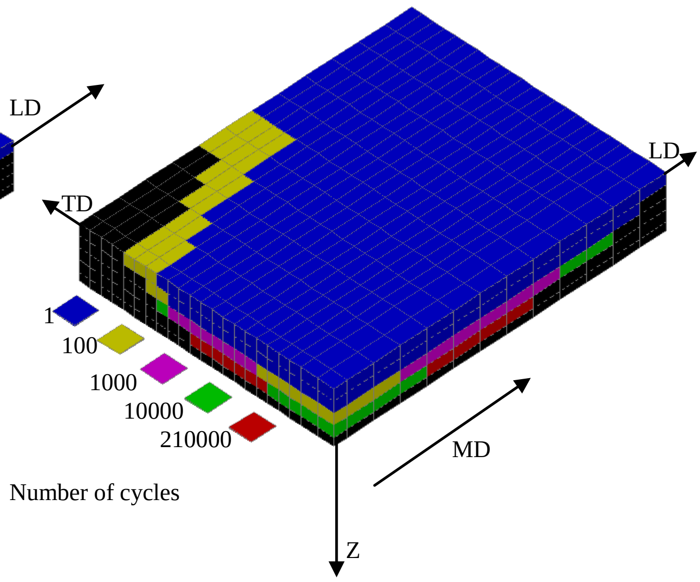

Fig. 8 Propagation of cracked element for slab with frost damage (bottom surface showed on top)

-ept by Li and Matsumoto[2]. As the load capacity with the current formed cracked elements cannot reach the moving load level, new cracked elements are required to reach the load level. This process is repeated till failure.

By taking advantage of symmetry, half of the studied slab is analyzed.

3. RC SLAB MODELING

The studied slab (**Fig.5**) is of dimension 2300 ×3000 × 180 mm and is supported by steel I-beams along its width. Frost damage is simulated in experimental study by reducing the slab thickness by 10 mm rather than freezing-thawing the slab[7]. On the other hand, in the present proposed finite element method (**Fig.6**), the slab is divided into 5 layers, and the top layer of 10 mm thickness is frost damaged. In this respect a FTC equivalent plastic strain of 0.782 is adopted in this study. The concrete and reinforcement properties of the slab are the same as those used in the experimental study as shown in **Table 2**.

4. RESULTS

4.1. Propagation of cracked elements

Fig. 7 and **Fig. 8** show the propagation of cracked elements under moving load at different numbers of cycles for both studied slabs.LD, TD and MD stand respectively for slab length direction, slab width direction and moving load direction. Due to the symmetry, one fourth of the slab model is analysed.

Black elements represent non-cracked zone. The cracked zone caused by the first cycle is coloured in blue while the cracked zones caused by further cycles are indicated by other coloured elements.

In both cases, it has to be noticed that, the cracked zone in slab length direction is larger than that in slab width direction. This is because the applied load is a load moving along the slab length. The size of the cracked zone after the first cycle is considered as an indication of the fatigue life of the studied slabs. Larger it is, shorter the fatigue lifeis. In fact cracked elements at the first cycle were subjected to crack closing and opening processes more than those at the further cycles.

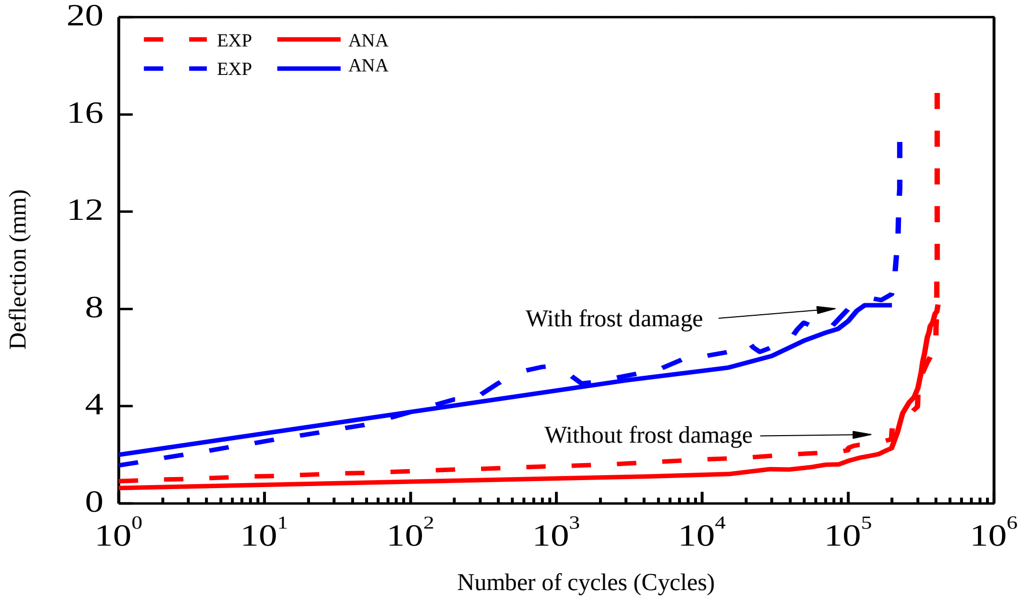

Fig. 9 Midspan displacement evolutions

Table 1. Applied load[7)]

Number of cycle N	Load (kN)
1 ≤ N ≤ 100000	130
100000 < N ≤ 200000	140
200000 < N ≤ 300000	170
300000 < N ≤ 400000	200
400000 < N	230

Table 2. Material properties[7)]

Materials	Concrete	Steel
Poisson's ratio	0.2	0.3
Strength (MPa)	40.1	235 (yield)
Young's modulus (MPa)	23460	200000

The slab without frost damage presents smaller cracked zone at first cycle than that with frost damage. This means that the slab without frost damage will need more cycles to propagate additional cracked elements until fatigue failure occurs compared with the slab with frost damage.

4.2. Midspan displacement evolution

Fig. 9 shows the midspan displacement evolutions obtained by experiment and numerical analysis for the slab without frost damage and that with 10 mm frost damage.

It comes out that the midspan displacement evolution for the RC slab with frost damage shows a larger slope than that without frost damage. A possible explanation for this is the reduction of the slab flexural stiffness due to the concrete stiffness deterioration caused by the frost damage.

The fatigue life of the RC slab with frost damage is shorter than that without frost damage consideration. This is due to irreversible tensile deformation and cracking during freezing and thawing cycles affecting the mechanical properties of concrete namely strength and stiffness.

Fig. 9 also shows that there is a good agreement between the numerical and experimental results.

5. CONCLUSION

The followin g conclusions can be drawn from this analytical approach:

1) The fatigue life prediction of frost damaged RC deck slab can be simulated through this numerical model.
2) The fatigue results are obtained and good correlation between the experiments and the developed numerical model exists.
3) Increasing of number of cycles under moving loading leads to a decreasing of crack bridging stress and an increasing of the cracked zone area more quickly for slab with frost damage than that without frost damage.

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